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ABSTRACT

Digraphs, graphs and task analysis were used to map out the content structure of a programed text (SMSG) in elementary probability; mathematical structure was operationally defined as the relationship between concepts within a set of abstract systems. The word association technique (WA) and paragraph construction technique (PC) were used to measure the existing relations (cognitive structure) in S's memory with respect to the probability theory present in the text. The purpose of this study was to measure the influence of content structure (mathematical structure) of the text on the subjects' cognitive structure. Control and experimental Ss (N=181) were sixth-grade, eighth-grade and high-school (grades 9-12) students. Experimental Ss read the probability text while the others read a programed text unrelated to probability. Ss were pre- and posttested and given retention tests. Results indicated that the experimental Ss' measured cognitive structure highly resembled the text's content structure following instruction. The WA and PC test also appeared to be useful for formative evaluation of the programed text and gave different information than did achievement tests. (JP)

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AN EXPLORATORY ANALYSIS OF CONTENT STRUCTURE AND COGNITIVE STRUCTURE IN THE CONTEXT OF A MATHEMATICS INSTRUCTIONAL UNIT

A DISSERTATION
SUBMITTED TO THE SCHOOL OF EDUCATION
AND THE COMMITTEE ON GRADUATE STUDIES
OF STANFORD UNIVERSITY
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

by
William Edward Geeslin
July 1973

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CHAPTER I

The Problem, Purposes of the Study, Overview of the Study

Identification of the Problem

During the past decade mathematics curricula have been revised significantly in an effort to provide students with a greater understanding of mathematics. Curriculum developers have attempted to communicate something more than algorithms and computational skills to the student (cf. Report of the Commission on Mathematics, 1959). That is, the student also is expected to learn relationships among mathematical concepts. In short, one purpose of the new curricula is to familiarize students with the structure of mathematics (Report of the Cambridge Conference, 1963).

This focus on mathematical structure led to the formation of several curriculum groups. They were charged with incorporating the structure of mathematics into the public school mathematics curricula. In discussing one of these groups, the School Mathematics Study Group, Begle (1971, p. 68) writes:

> ...by paying careful attention to the structure of mathematics, the way mathematical ideas fit together, rather than relying on intricate and ingenious computations, it was possible to solve difficult and important mathematical problems... . The importance of this change of emphasis from ingenious computations to basic concepts and the structure of mathematics gradually became clear.

In spite of the emphasis in past years on the structure of mathematics, very little empirical work has been done concerning the communication of mathematical structure to the student. One possible reason for this paucity of research is that mathematical structure has not been defined operationally. Another possible reason is that the thrust of curriculum revision has been toward development rather than research and evaluation. Only recently have a few, systematic, empirical studies in mathematics education (e.g. the National Longitudinal Study of Mathematical Abilities, School Mathematics Study Group) been carried out. If learning mathematical structure is as important to mathematics education as leaders in the field suggest, a definition of structure which leads to empirical methods for studying structure are critical to the improvement of mathematics curricula.

The purpose of this study, in broad terms, is to define what is meant by structure in mathematics curricula and to investigate some methods for examining structure in the curriculum and structure in the student's memory after instruction. Of peripheral focus in this study is the possible usefulness of information gathered by the various procedures for curriculum evaluation, both formative and summative (Scriven, 1967). The study itself is not a curriculum evaluation, but if the instruments and procedures used in the study yield information concerning the representation, communication and learning of mathematical structure then the instruments also should lend themselves to evaluation.

Definition of the Research Problem

Begle (in preparation) states, "We consider mathematics to be a set of interrelated, abstract, symbolic systems."

He emphasizes that mathematical structure is a combination of within system relationships and between system relationships. Shavelson (1970, p. 1) used a similar general definition for structure.

> ...structure is defined as an assemblage of identifiable elements and the relationships between those elements. Structure may be objective and real or internal and subjective. In interpreting literature on structure, special attention will be paid to identifying elements and stating how they are interrelated.

Begle's discussion and Shavelson's definition are sufficiently similar to indicate that Shavelson's work may be relevant to the representation of mathematical structure.

For the purposes of this study, mathematical structure is defined to be the relationships between concepts within a set of abstract systems. (Concepts may be represented by either symbols or words.) Suppose we could get measures of structure in both the mathematics curriculum and in a student's memory after learning from the curriculum which are consistent with our definition of mathematical structure. Comparison of the two representations of structure might provide some insight into the extent to which the goal of teaching mathematical structure has been achieved.

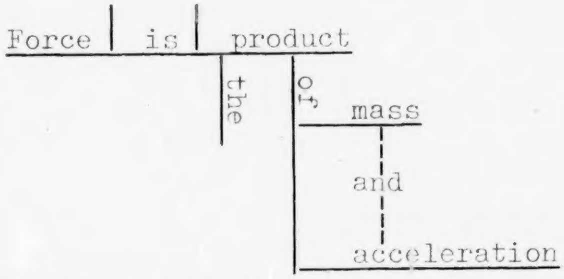
Content structure. When we speak of the structure presented by a mathematics curriculum we refer to what Shavelson (1970) has termed content structure. Content structure is "the web of facts (words, concepts), and their interrelations in a body of instructional material [Shavelson, 1970, p. 9]." The problem, then, is to identify a method for mapping the concepts and their interrelations in a mathematics curriculum. One possible method, applied by Shavelson (1970), is the theory of directed graphs. "This theory is concerned with patterns of relationships among pairs of abstract elements [Harary, Norman, and Cartwright, 1965, p. 2]." The theory of directed graphs, or more briefly, digraph theory,

> ...deals with abstract configurations called digraphs, which consist of 'points' and 'directed lines.' When these terms are given concrete referents, digraphs serve as mathematical models of empirical structures, and properties of digraphs reflect structural properties of the empirical world... [Harary, et. al., 1965, p. v].

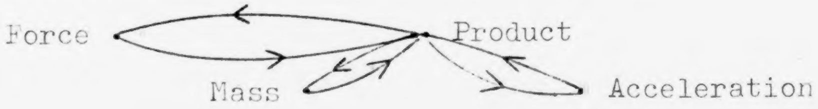
If we allow concepts (words or symbols) to be represented by points and their interrelationships to be represented by lines, then all true statements about the obtained digraph are correspondly true of the empirical world. Digraph theory, then, provides a method for transferring written instruction into a structural representation consistent with our definition of mathematical structure.

Kopstein and Hanrieder (1966) and Shavelson (1970) have applied digraph theory to the analysis of content structure. Shavelson (1970) developed one possible set of rules for transferring prose into a digraph. He used the sentence as a unit of analysis and separated it into its syntactical components. He then gave rules for relating various components of the sentence to the digraph. Shavelson (1970, p. 37) gave the following example from physics:

...'force is the product of mass and acceleration' and was diagrammed as:



...The following digraph for F = MA resulted:



As Shavelson (1970) points out, digraph theory is only one possible way of representing the structure presented by text (see Berelson (1954) for a review of other alternatives). An equally plausible method for representing mathematical structure would be the use of graph theory (Harary & Norman, 1953). Graph theory differs from digraph theory in that non-directed lines are used. A third method for representing content structure is task analysis (cf. Gagne, 1965). This results in a logical hierarchy of concepts. Other alternatives, such as transformational grammar (Chomsky, 1965), might be used. In this study, three methods of analyzing content structure--digraph theory, graph theory, and task analysis--will be investigated.

Cognitive structure. Curriculum evaluators have used numerous methods in attempting to decide whether or not the goals of the curriculum have been achieved. Most evaluation instruments rely on achievement tests to assess the success of a curriculum. However, the fact that a student can add and subtract does not imply necessarily that the student is familiar with the relationship between addition and subtraction. Various tests have been developed to determine if a student understands these relationships. For example, the National Longitudinal Study of Mathematical Abilities (NLSMA) developed tests for four cognitive levels (Romberg & Wilson, 1969). The tests actually do not attempt to measure the learning of mathematical structure, but purport to measure complexity levels of mental activities (Romberg & Wilson, 1969). If the curriculum developer or mathematics educator actually desires the student to be familiar with a structure in mathematics, this objective needs to be defined in terms suitable for empirical investigation. Then the evaluator should determine the success of the curriculum in attaining this objective. "When we take so much care to develop understanding and creativity in the student, it would be a pity to test his achievement only in terms of the mechanical skills and rote responses he has learned. [Cambridge Conference Report, 1963]."

When we speak of structure in a student's memory, we refer to "cognitive structure." Cognitive structure is a "hypothetical construct referring to the organization (interrelationships) of concepts in long-term memory [Shavelson, 1970, p. 9]." One method for examining cognitive structure is the technique of word association (WA) (cf. Deese, 1962, 1965; Johnson, 1964, 1965, 1967, 1969; Shavelson, 1970). With this method, the student is presented a concept in mathematics, for example, and asked to call forth as many other related mathematical concepts as he can. The rationale for using this method to examine cognitive structure is given by Deese (1962, p. 174): "associations derive in whole or part from the structures or categories of the human mind." According to Deese (1962, 1965) and Shavelson (1970), the meaning of words is defined, in a large part, by the organized relations among words. Johnson (1967) concurs that learning subject matter is, in part, internalizing relationships between concepts.

One way to examine the organization of concepts in a student's memory is to compare the overlap in responses to various concepts. "The underlying assumption is that the order of response retrieval from long-term memory reflects at least a significant part of the structure within and between concepts [Shavelson, 1970, p. 6]." Since the problem at hand is to obtain a representation of the student's cognitive structure concerning mathematics, we are interested in how the student organizes the mathematical concepts. The WA technique reveals something about that organization. Deese (1962) took concepts which had an underlying categoric structure, collected WA data, and was able to retrieve an interpretable, logically consistent structure from the WA data. Thus the WA method may be appropriate for investigating that portion of a student's cognitive structure concerning mathematics.

Rothkopf and Thurner (1970, p. 83) observed that "the performance changes that result from experience in the verbal learning laboratory may [...] be quite untypical of the manner in which verbal performance is ordinarily modified by verbal experiences in man." They suggest that we should investigate quantitative indicators that resemble more closely normal language usage. This line of reasoning led Rothkopf and Thurner to suggest a second method of investigating cognitive structure, namely the use of essay protocols.

> Techniques for the analysis of essay protocols are also applicable to the analysis of instructional text. As such they offer the possibility of providing quantitative indicators of instructive experience and more powerful and realistic characterization of independent variables in instruction. [Rothkopf & Thurner, 1970, pp. 88-89].

Since the word association technique is not the only possible (and perhaps not the best) measure of the learning of mathematical structure, a second measure of cognitive structure will be used in this study. Following a literature search for similar instruments and discussions with advisors[1], two new instruments, a paragraph construction task and a sentence construction task[2], were used to measure learning of mathematical structure.

With the paragraph construction (PC) test, students write a paragraph explaining the mathematical relationship(s) between key concepts. Although Rothkopf and Thurner (1970) asked Ss to write about only one concept at a time, the emphasis of the present study is on the relationship between concepts and thus it was felt that the instructions to explain the relationship between two concepts corresponded more closely to our definition of structure than did the method of Rothkopf and Thurner.

---

[1] The author is grateful to Professor Lee J. Cronbach for his suggestions and criticisms in this area.

[2] Results of a pilot study indicated the sentence construction task was too constrained a task for Ss. Sentences were often nonsensical or inappropriate, e.g. "Probability and event are both nouns." Ss who did respond appropriately to the sentence construction task usually responded in a similar but expanded manner on the paragraph construction task. Thus, in general, the useful information from the sentence task was obtained also from the paragraph task. Therefore, it was decided not to use the sentence construction task in this study.

Overview of the Study. The purpose of this study was to examine the communication of the mathematical structure of a programmed text in probability to Ss. Ss, chosen from three school levels (elementary school, junior high school, and high school), were assigned randomly within each school level to experimental and control groups. Ss received instruction in their regular classrooms[3]; experimental Ss read the probability text, while control Ss read a programmed text on a different mathematical topic. Prior to and following instruction, Ss received tests on achievement, attitude, and cognitive structure.

Cognitive structure was investigated using WA and PC techniques. Digraphs, graphs, and task analysis were used to represent content structure. The various representations of content structure and cognitive structure were compared. Ss learning of mathematical structure was compared to achievement, attitude, and, in some cases, ability data.

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[3] At the high school level, it was necessary to remove some Ss from their regular classrooms.

CHAPTER II

Review of the Literature

Mathematics educators have put an increasingly strong emphasis on communicating the structure of mathematics to the learner, particularly in the public schools (Report of the Commission on Mathematics, 1959). Brown (1971) in discussing the changes in the mathematics curriculum over the past few years states:

> we study numbers themselves [and] reflect the nature of mathematics as a discipline. Mathematicians refer to it as structure. We are studying the basic structure of mathematics [and in] pursuit of properties that reveal the underlying nature of the mathematical discipline...

Begle (in preparation, Chapter III) claims

> A prerequisite to a study of the learning of mathematics is a clear understanding of the nature of the mathematics to be learned. We consider mathematics to be a set of _interrelated_, _abstract_, _symbolic_ _systems_.

Begle goes on to say

> Thus the structure of mathematics has two parts. On the one hand, each mathematical system has its own internal structure. On the other hand, there are linkages between different systems which also contribute to the structure of mathematics.

Schwab (1962) argues that the structure of the curriculum should represent the structure of the discipline and it is this structure that we are attempting to communicate to the student. Scott (1965) in studying the organization of text concluded that the academician does put structure in curriculum and that empirical factors represent this structure fairly well. However, Brumfiel (1971) in discussing mental associations indicates that different mathematicians may see particular aspects of mathematical structure differently.

The consensus, then, is that the curriculum should attempt to communicate structure to the student. This structure may vary depending on the particular curriculum and the structure is subject to empirical investigation.

The present study is an empirical investigation of the communication of a mathematical structure using methods developed by educational psychologists (cf. Deese, 1962; Gagne, 1970; Rothkopf & Thurner, 1970; Shavelson, 1970). Studies concerning the learning of structure have concentrated on one of three basic elements: concepts, competencies, or algorithms. Although the present study concentrates on concepts and their interrelationships, the literature search in mathematics education revealed the other two alternative approaches to be the only empirical efforts in studying mathematical structure. Therefore, this chapter briefly reviews the alternative approaches followed by a review of the approach used in the present study.

Alternate Approaches to Investigating Structure

Organization of competencies. Gagne (1962, 1965, 1970) discusses the logical analysis of content structure or task analysis. The focus of Gagne's suggestions is on competencies rather than concepts. The task analysis proceeds by deciding on the final competency(s) expected of students after instruction and then logically determining all subordinate competencies that are necessary for carrying out the final task(s). Gagne (1962) states that the task of a learning program is to: (1) insure high recallability of relevant learning sets on which achievement has been demonstrated; (2) making possible identification of expected performance and of new stimuli, for each newly presented task; and (3) guiding thinking so as to suggest proper directions for hypotheses associating subordinate learning sets with each new one. Gagne and Paradise (1961) present a study which lends support to the theory that differences in rate of completion of a learning program are primarily dependent upon the number and kind of learning sets the learner brings to the situation, and only secondarily upon his standing with respect to certain basic abilities.

Regnier and de Montmollin (1968) used graph theory to represent content structure as described by Gagne (1962, 1965). That is, points on the graph represent competencies. Thus a method for obtaining a graph from a logical hierarchy of competencies is given, but this graph is not directly comparable to a graph which maps the organization of concepts. Thurner and Johnson (1970) discuss the logical configuration of concepts.

Algorithmic approaches. A few researchers in mathematics education have been interested in the learning of mathematical structure. These authors have defined structure in a different manner from the present study but are noted as alternatives to the present procedures.

Dienes and Jeeves (1965, 1970), Branca (1971), and Branca and Kilpatrick (1972) have developed one method for investigating learning of mathematical structures. Branca (1971) was concerned with Ss' strategies for learning structure. Ss were asked to discover the rules of a game by trial and error. The structure of the game was the Klein group structure. The results are not pertinent to the present study, but Branca's implicit definition of structure is important. S was said to have learned the structure when he had correctly determined the rules of the game and verbalized these rules in a manner consistent with the way the rules would be expressed in mathematics. This notion of structure departs from either Shavelson's (1970) definition concerning relationships between concepts or Gagne's (1962) reference to logical structure of competencies.

Scandura (1971) outlines his basic theory of structural learning developed through a series of empirical and theoretical studies. He "proposes and defends [...] that rules are the basic building blocks of all mathematical knowledge and that, if looked at the right way, all mathematical behavior is rule governed. [Scandura, 1971, p. 184]." Scandura (1971, p. 186) uses "rule" in a different sense than Branca:

> a class of behavior is said to be [rule governed] if the behavior can be generated by a common algorithmic (generative) procedure of some sort...a person who has mastered any underlying procedure should [...] be able to generate each and every response, given any particular stimulus in the class of stimuli.

While Scandura refers to linguistic theories, he appears to derive his "rules" subjectively rather than empirically. Secondly, though this set of rules may explain behavior it does not appear to be reducable to relationships between concepts, particularly since Scandura emphasizes process rather than a static entity of the type the present study is concerned with in terms of structure.

Mayer and Greeno (1970) hypothesized that different instructional procedures might produce qualitatively different learning outcomes. "The concept of binomial probability was taught using a method that emphasized calculating with the formula, and a method that emphasized the meanings of the variables in the formula [Mayer & Greeno, 1972, p. 165]." Results of three experiments indicated there was not a significant difference between treatments in terms of the total transfer test score. However, the transfer test was divided into four parts: familiar items, transformed items, unanswerable items, and general questions. "Large interactions in transfer performance were obtained in three cases, indicating that the two methods produced structurally different learning outcomes [p. 165]." This study indicates content structure may have an effect on performance tests as well as cognitive structure measures.

Concept Approach to the Learning of Structure

The concept approach to structure is presented in detail by other authors (cf. Gagne, 1962; Deese, 1962, 1967; Anderson, 1969; Fillenbaum & Rapoport, 1971; Shavelson, 1970). Therefore this review will briefly present only the main points.

Content structure. The literature concerning content structure may be divided into two categories depending on whether the emphasis is on the organization of concepts or competencies. Structure emphasizing concepts is most relevant to this study, however, some attention is given above to organization of competencies as an alternate method since this is more common in mathematics education.

Kopstein and Hanrieder (1966) carried out one of the first studies which suggested using directed graphs (or digraphs) (cf. Harary, et. al., 1965) to represent content structure. Points on the digraph represented concepts and lines between points indicated relationships between concepts. Although the purpose of their study--to investigate the strength or vulnerability (cf. Harary, et. al., 1965) of the resultant digraph--is not relevant to the present study, their suggestion of transforming content into a digraph is most important.

Kingsley, Kopstein, and Seidel (1969, p. 3) discuss the use of graphs to represent content structure:

> The requirement exists for a metalanguage in which to describe communicable knowledge. A strong candidate for this role is the mathematics of nets and graphs...It will be readily apparent that such a representation amounts to a 'map' of a knowledge space.

Shavelson (1970) reviews the literature concerning the use of digraphs to represent content structure. Additionally, he gives the rules for transforming each sentence in the content to a digraph (see Shavelson, 1970, Appendix A) and shows how to combine the separate digraphs into a super-digraph which represents the total content structure presented by a text. Shavelson's methods are used in the present study.

Other studies (cf. Johnson, 1965, 1967, 1969; Frase, 1969) used digraphs to represent content structure, but do not develop procedures for mapping the instructional material with a digraph as did Shavelson. Frase and Silbiger (1970) and Frase (1970) discuss extensions of the use of digraphs to include sequence of presentation. Anderson (1969) presents a theoretical discussion of how these procedures might be extended to include the structure of teacher presentation. These extensions of the use of digraphs are not used in the present study.

Cognitive structure. Deese (1962) explains that the meaningfulness of words refers to organized relations among the words and among the words and objects in the natural world. Ausubel and Fitzgerald (1961) refer to the structure in memory as an ideational scaffolding. That is, a person stores concepts in memory in an organized manner. Lippman (1971) discusses the development of this organization in memory and studied the difference in organization due to age. Lippman concluded that a shift in type of organization occurs near the age of seven. Bruner (1960, p. 7) has proposed that "Grasping the structure of a subject is understanding it in a way that permits many things to be related to it meaningfully. To learn structure in short is to learn how things are related." Anderson (1969, p. 8) indicates

> [static] structure is the production of multiple associations among units of information and the presentation of logical thinking statements which interrelate them. It includes organized response patterns such as classifications, concepts, and principles which approximate rigid (static) associations...

Fillenbaum and Rapoport (1971, p. 1) connect this structure to linguistics:

> Relational systems in linguistics are characterized by their elements and the types of relations holding among them. The syntactic system generates strings of minimal syntactically functioning elements and specifies the structural interrelationships among them.

Shavelson (1970, p. 1) defines structure "as an assemblage of identifiable elements and the relationships between those elements." We concur with Shavelson's definition and approach the study of structure on the basis outlined above.

Empirical studies using the word association technique. This section outlines prior studies which have used WA techniques to assess cognitive structure, particularly those studies which use the WA test as a learning measure. The word association technique was used to investigate cognitive structure in the present study. Noble (1963) proposed this method as a measure of meaningfulness. Deese (1965) and Dixon and Horton (1968) reviewed the research on associations. Shavelson (1972) presents an argument for this measure's relationship to learning.

Building on the proposal by Noble (1963), several studies have used the WA technique to examine learning. In a series of studies by Johnson (1965, 1967, 1969), the number of word associates was correlated with the number of problems solved in a study on the learning of a short unit on physics. He was not able to show that responses on the association test were related uniformly to problem-solving success in the same way either for all the words on a single association test or for the same word on two association tests, but he showed certain important relationships. Johnson (1967) concluded that words were more meaningful for high achievers than low achievers and also that meaningfulness is related to the frequency of occurrence in text. Johnson (1969) claimed that there was a significant increase in the numbers of responses to a WA test after instruction as compared to before instruction. Johnson also discussed the fact that concepts have a meaning both within and without a subject and thus the kind of structure in memory may not change so much as the quality of structure.

Shavelson (1970) used the WA technique to measure learning of physics structure. He found significant changes during instruction in instruction Ss' responses to the WA test as well as significant differences between treatment groups after instruction.

Rothkopf and Thurner (1970) used the same instructional material as Johnson and Shavelson and found a high correlation between WA responses and essay responses after instruction. Lambert (1970) claimed that both SES and ability are related to paired associate learning tasks.

In summary, as argued by Shavelson (1970), responses to the WA test not only reflect the cognitive structure in memory, but also reflect learning.

Comparisons of cognitive structure with content structure. Shavelson (1970) appears to be the first author to compare an empirical representation of content structure to an empirical representation of cognitive structure. Shavelson reviews studies leading to the components (cognitive structure and content structure) and these studies are briefly noted above. Therefore, this section will discuss Shavelson's study only as it is the basis for the present study.

Shavelson used physics material as instructional material--the same material which was used by Johnson (1965, 1967, 1969) and Rothkopf and Thurner (1970). Ss (N = 40) were paid volunteer high school students (grades 10-12) who had not taken a high school physics course. Ss were divided randomly into instruction (N = 28) and control (N = 12) groups. All Ss received aptitude, achievement, and WA tests as pretests. Instruction Ss read five sections of physics material--one section per day, each day being a two hour period--and responded to a WA test at the end of each instructional period. Control Ss took only the WA tests and did so in a smaller number of days. All Ss received an achievement posttest.

Table 1

Euclidean Distance Matrix: The Distance Between Content Structure and Cognitive Structure for Instruction and Control Groups Across the Six Test Days

DAY GROUP	1	2	3	4	5	6
INSTRUCTION	6.49	5.92	5.53	4.90	4.52	4.22
CONTROL	6.69	6.28	6.07	6.17	6.17	6.25

Shavelson's results indicated the instruction Ss did significantly better on the achievement posttest ($p < .05$) than control Ss and that instruction Ss performed significantly higher ($p < .05$) at post time than at pretest. That is, instructional Ss learned the material to a significant degree in terms of solving physics problems.

The WA tests were administered in a repeated measures design and results indicated that control Ss maintained a stable cognitive structure throughout while instruction Ss showed a variable cognitive structure presumably due to instruction (see Table 1 from Shavelson, 1970, p. 83). Shavelson also noted that instruction Ss' cognitive structures tended to move toward the content structure as they received instruction. Instructional Ss' cognitive structures did not change significantly in terms of configuration of concepts, but changed in a qualitative manner. Shavelson claimed this was due to the fact that ordinary usage of the physics concepts had an influence on cognitive structure and, in fact, the physics instruction might actually reenforce certain associations. In line with these conclusions, Shavelson found an increase in WA response frequency by instructional Ss which was not the case for control Ss.

Shavelson was not able to show a uniformly significant correlation between WA data and either aptitude or achievement data:

> To summarize, verbal ability plays a decreasing role in association generation across instruction days. But it is an important predictor of achievement. Abstract reasoning ability[...] plays an increasingly important role during learning[...]and is an important predictor of posttest achievement. This finding supports the interpretation that for Ss who performed well in solving problems at the end of instruction (high posttest achievement), the concepts became more meaningful earlier.

> Subsequently, these Ss were able to "chunk" information in the form of equations. Having these equations readily available in memory enabled them to solve physics problems on the posttest on achievement more effectively. [Shavelson, 1970, p. 106]

CHAPTER III

Method

Instructional Material

The text used by the experimental group was an introduction to probability theory and was developed under the direction of the School Mathematics Study Group (SMSG).[1] The topic of probability was chosen because: (a) it could be presented by text alone; (b) it was justifiable as important mathematics for Ss to learn[2]; (c) it could be placed easily at most points in the regular curriculum sequence; (d) it assumed a minimum number of mathematical concepts and skills and thus was appropriate for a wide range of grade levels[3]; and, (e) it was unfamiliar to the majority of K-12 students in the Stanford University area.

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[1] The author wishes to express his appreciation for the help of Stanley Pogrow and Robert Wise in writing the text. (The entire text is available from the ERIC Science, Mathematics and Environmental Educational Clearinghouse, Columbus, Ohio.)

[2] For example, the School Mathematics Study Group argued:

> Some understanding of probability and statistics is essential for the educated citizen in modern society. [...] Probability theory is a requisite for the techniques of statistical analysis and statistical inference that play so large a role in industry, government, economics, social science, and all branches of physical and biological science [School Mathematics Study Group, 1971].

[3] A series of unpublished formative evaluation studies by the author and a study by McLeod (1971), have shown that grade five students are capable of learning the material, while at the same time high school students do not find the material to be at too low a level to keep their interest.

A programmed instruction format--small steps, constructed responses, and continual feedback on the correct responses--was used. The format was selected for several reasons. It minimized the number of substantive questions asked by students and thus minimized the chance that proctors would "teach" a structure different from the text structure by answering students' questions. And it permitted an examination of the applicability of the structure methodology to students of various ages and ability levels.

The probability text is divided into three sections of approximately seventy pages each. It assumes the students have an intuitive idea of prediction, chance, and experiment. Section 1 of the text covers the concepts of "probability," "equally likely," "outcome," "event," "experiment," and "zero." After completing Section 1, the student should be able to list the outcomes of a simple experiment such as tossing a single die. Secondly, he should be able to determine which outcomes form an event, e.g. number greater than 3, and find the probability of this event. Section 2 adds the concept of "trial" and expands the concepts in Section 1 to more complicated experiments. Upon completion of Section 2, the student should be able to distinguish between a trial and an outcome; list the outcomes in an experiment such as spinning two spinners; determine which outcomes form an event, e.g. same color on both spinners; and find the probability of this event. Section 3 covers the concepts of "independent," "intersection," and "mutually exclusive" and expands previous concepts to experiments such as flipping a coin three times. Upon completion of Section 3, the student should be able to: (a) determine whether two events are mutually exclusive or independent; (b) find the probability of an event concerning an experiment such as spinning a spinner twice; and, (c) find probabilities of events involving drawing marbles with or without replacement. With the exception of a few frames toward the end of the text which require the multiplication of simple fractions, S should be able to rely entirely on his ability to count in obtaining correct responses to frames requiring numerical answers.

A typical frame of the probability text is shown in Figure 1. In general a frame consisted of a short piece of prose followed by two questions. The student wrote his answer to each question in the blank provided. At the top of the page immediately following the frame the correct answers to the questions were provided. A horizontal line was drawn across the page to separate the answers at the top from the new frame below.

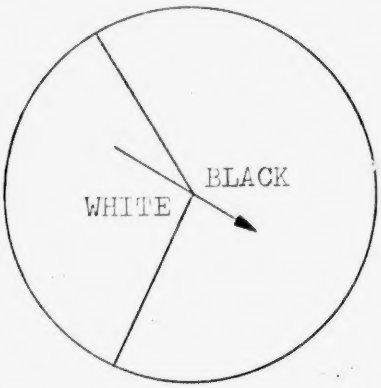


Figure 1. A typical Frame from the Probability Text

For the experiment of spinning this spinner, we say that Black and White are not equally likely outcomes.

1) Are the outcomes of the experiment of tossing a coin equally likely? ________________

2) Are the outcomes of the experiment of throwing a die equally likely? ________________

Representation of Content Structure

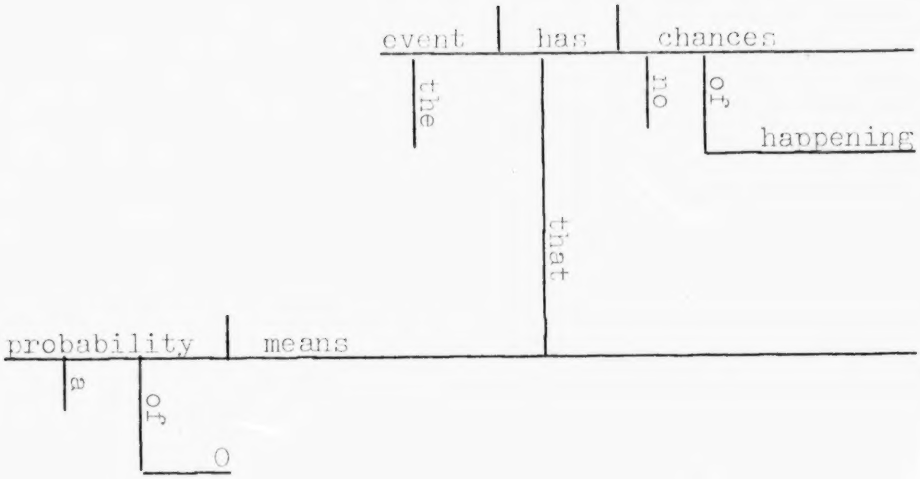
Content structure was defined as the web of facts (words, concepts) and their interrelations in a body of instructional material. In order to apply this definition of content structure to the probability text, ten key concepts were selected for analysis: "probability," "equally likely," "outcome," "event," "experiment," "zero," "intersection," "trial," "independent," and "mutually exclusive." The key concepts were selected _a priori_ as being the most important, in a mathematical sense, in the text. That is, the text was designed specifically to teach these concepts and these concepts were thought to be crucial in the students' mastery of the instructional material.

The key concepts appeared to vary in two conceptual dimensions, as judged by the author. Some stimulus concepts are more "concrete" than others, i.e., they vary in the ease with which they can be represented physically. For example, "zero" may be considered more concrete than "independent." Secondly, the concepts varied in their familiarity to students, i.e., students are likely to have encountered the concept of "zero" more often than the concept of "mutually exclusive."

Three alternative methods for representing content structure were used: digraph, graph, and task analysis. The latter alternative is focused on relationships between competencies rather than concepts, while the digraph and graph use concepts as elements.

Digraph representation. One method for representing structure in the instructional material, termed "content structure," is the theory of directed graphs (Harary, et. al., 1965). (See Shavelson, 1970, Appendix A, or Shavelson, 1971, for a summary of the theory of directed graphs.) The steps followed in a digraph analysis are described fully by Shavelson (1970, pp. 35-40; see also Shavelson, 1972) and thus only a brief discussion need be presented here. The probability concepts were represented by points on the digraph and relationships between concepts were represented by directed lines connecting points. As noted in Chapter II, the theory of directed graphs is an abstract mathematical theory of structure in which structure is defined as points and directed lines. If the correspondence between digraph theory and the empirical world is accurate, then all true statements about the digraph are also true of the empirical world. The resultant digraph is considered to be one representation of the content structure.

Following Shavelson's (1970) procedures, all sentences in the text which contained at least two of the key concepts were selected for the analysis. The reason for selecting this set of sentences is that we were interested in the way pairs of concepts are interrelated in the text, and the sentence is our unit of analysis. For example, the sentence "A probability of 0 means that the event has no chances of happening" was selected because the concepts "probability," "0" (zero), and "event" were contained in the sentence. Each sentence containing two or more key concepts was diagrammed using a parsing grammar (Warriner and Griffith, 1957). For our example we obtained the following diagram:

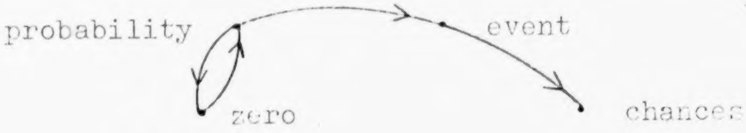

event
has
chances
the
no
of
happening
that
probability
means
a
of
0

The diagram was converted to a digraph using Shavelson's rules.

For example, one rule is:

> a preposition is a word used to show the relation of a noun or pronoun to some other word in the sentence. A preposition specifies a relation between two points on a digraph and is represented by a line. If the preposition gives direction ("to"), the relation is asymetric; if the preposition does not specify direction ("of"), the relation is symmetric.
>
> A group of words may act as a preposition: on account of, in spite of, divided by. [Shavelson, 1970, p. 140].

The digraph resulting from our example is:


probability
event
zero
chances

From the individual digraphs, a super-digraph was constructed incorporating the information from the single digraphs. Finally, a "distance" matrix was created in which each entry represented the minimum number of lines connecting any pair of key concepts. For our example we obtained[4]:

$$\begin{array}{c|cccc}
 & p & e & c & z \\
\hline
p & 0 & 1 & 2 & 1 \\
e & \infty & 0 & 1 & \infty \\
c & \infty & \infty & 0 & \infty \\
z & 1 & 2 & 3 & 0 \\
\hline
\end{array}$$

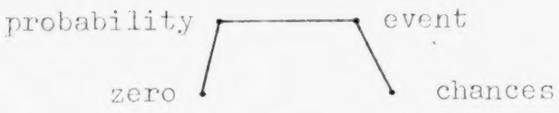
where: p = probability,
e = event,
c = chances, and
z = zero

Graph representation. The content structure or structure in the instructional material also was mapped with graph theory (Harary and Norman, 1953). Graph theory may be distinguished from digraph theory in that the former ignores the direction of lines while the latter places an emphasis on directed lines. The resultant graph was considered a second representation of content structure. The same key concepts were used in this analysis as were used in the digraph analysis. A symmetric distance matrix is obtained from the graph analysis. The elements in the graph distance matrix are equal to the smallest element in each pair of corresponding cells in the digraph distance matrix. Only a slight modification of digraph procedures is necessary to construct the graph. Thus, in the example presented in the digraph section above, the same sentences are selected for analysis, and the same diagram results. However, the graph and distance matrix are changed as shown below.

---

[4] In the actual analysis, the distance matrix is computed only for the super-digraph. However, this example demonstrates the connection between the digraph and the distance matrix.

1. Graph:



2. Distance Matrix:

	p	e	c	z
p	0	1	2	1
e	1	0	1	2
c	2	1	0	3
z	1	2	3	0

Note that the graph distance matrix will always be symmetric while this is not necessarily true of the digraph distance matrix. Obviously, if a symmetric digraph results from the digraph analysis, the structure representations by graph and digraph will be equivalent.

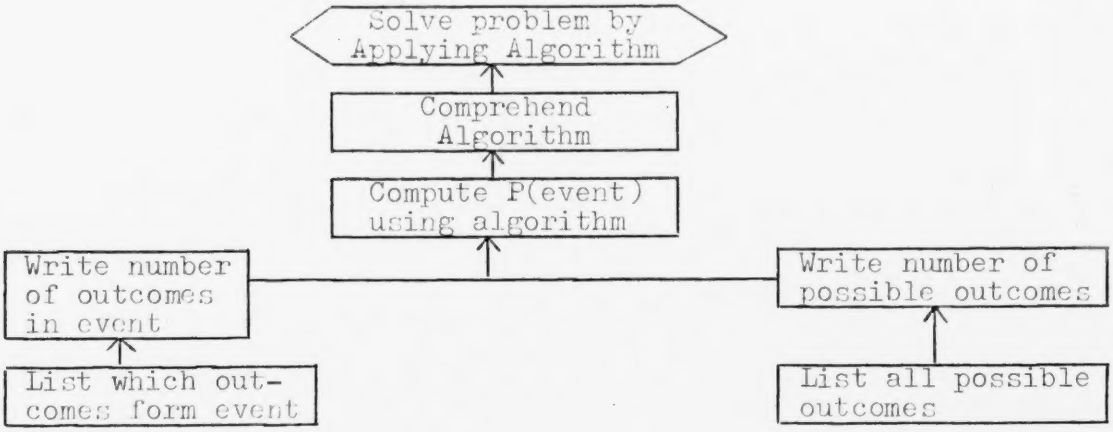
Task analysis representation. Finally, task analysis was used to map the structure of the instructional material (Gagne, 1965, 1970). Task analysis produces an alternate (to the digraph/graph analyses) structural representation. Points represent competencies and lines represent relationships between competencies. This is a psychological definition of structure and therefore different from what subject-matter experts mean when they use the term structure. However, we use task analysis in the present study to link the digraph/graph representations to a more traditional approach.

A task analysis works backwards from the final outcome(s) of instruction, in a logical manner describing all the necessary prerequisites that the student needs in order to exhibit a satisfactory performance on the final outcome(s), to the initial competencies the students are assumed to possess. This procedure results in a hierarchical flow chart which maps the instructional sequence (to some extent) and the psychological/ logical structure of the text.

For example, after completing Section 1 of the probability text, the student should be able to compute the probability of the event "number greater than 3" for the experiment of tossing a fair die. In order to do this he must be able to apply and comprehend the algorithm for computing probabilities. To do this he must be able to determine the number of outcomes in the event "number greater than 3," determine which outcomes are in the event, determine the number of possible outcomes, etc.. This portion of the task analysis is shown in Figure 2.

Figure 2

Sample Task Analysis


Solve problem by Applying Algorithm
Comprehend Algorithm
Compute P(event) using algorithm
Write number of outcomes in event
Write number of possible outcomes
List which outcomes form event
List all possible outcomes

Subjects

Students from an elementary school (Grade 6), a junior high school (Grade 8), and a senior high school (Grades 9-12) served as subjects (Ss) in the study. No S had received prior formal instruction in probability.

Since the experiment was conducted at the end of the school year and over approximately a four week period, several Ss were deleted altogether or from certain analyses due to absenteeism, withdrawal from school, or conflicts with other school activities. In this section the number of subjects associated with each school level reflects the number of Ss who attended at least one test session.

Grade six subjects. Grade six Ss (N = 59) were taken from two intact classes in one elementary school in Santa Clara County, California. Ss were assigned at random to experimental (N = 33) and control (N = 26) groups. Ss varied widely in ability, Lorge-Thorndike IQ's ranged from 68 to 143. The school principal reported that Ss varied in socio-economic status (SES), ethnic background and motivation; and that several Ss were performing well below expectations. A few Ss had severe reading deficiencies. In general, most Ss in the sample may be described as low-middle to middle SES, near average in ability, and Caucasian.

Grade eight subjects. Grade eight Ss (N = 87) were taken from three intact classes in one junior high school in the same district as the elementary school Ss. Ss were divided randomly into control (N = 44) and experimental (N = 43) groups. The principal indicated that these Ss were average and slightly above average in mathematical ability. No severe reading difficulties were noticed by the experimenter. Only a few Ss showed low motivation; this impression was confirmed by the mathematics teacher. The only ability data made available to the experimenter were scores on the Minimum Essentials of Modern Mathematics test and these scores ranged from 11% to 99.5% correct.

High school subjects. The third group of Ss (N = 34) were volunteers recruited from study halls and mathematics classes in one high school in San Mateo County, California. Ss were ninth (N = 13), tenth (N = 3), eleventh (N = 10), and twelfth (N = 8) graders. Ss were divided randomly into experimental (N = 20) and control (N = 14) groups. The principal described the Ss as middle and upper-middle SES, Caucasian with varied mathematical ability and background. Ability data were not available for these Ss.

All Ss had completed a ninth grade mathematics course, many had completed some algebra and geometry, and one S had completed a trignometry course. Many Ss were not taking a mathematics course at the time of the experiment. The remaining Ss were enrolled in a variety of mathematics courses.

Treatments

The experimental treatment consisted of instruction from the programmed text on probability theory. Subjects in the control group read a programmed text on a mathematical topic unrelated to probability. At each grade level, the control group's text was similar in format and outward appearance to the probability text. The texts used for control Ss had no mathematics in common with the probability text.

Ss in the sixth and eighth grade control groups read a programmed text on factors and prime numbers.[5] It was divided into five sections and covered the basic concepts of composite and prime numbers. The text developed a division algorithm for testing whether a number was prime. Multiples of a number and least common multiples of pairs of numbers were covered. Each section was followed by a criterion test.

Subjects in the high school control group read a programmed text on negative number bases.[6] It reviewed positive number bases and computation in positive bases other than base ten. The text was divided into three sections: (a) polynomial representations of numbers, (b) negative number bases including addition and subtraction of numbers in negative bases, and (c) representation of negative numbers in negative number bases.

Instrumentation

Attitude questionnaire. The attitude questionnaire was the "Pro-Math Composite" scale (PY011; see Wilson, Cahen, and Begle, 1968) developed by the National Longitudinal Study of Mathematical Abilities (NLSMA). The scale was designed to measure general attitude toward mathematics. An example item is

---

[5] The factors and prime numbers text was developed for SMSG by J. W. Green. (Available from the ERIC Science, Mathematics, and Environmental Education Clearinghouse, Columbus, Ohio.)

[6] The negative number bases text was developed for SMSG by Norman Webb. (Available from the ERIC Science, Mathematics, and Environmental Clearinghouse, Columbus, Ohio.)

> I can get along perfectly well in everyday life without mathematics:
>
> (A) strongly agree, (B) agree, (C) don't know, (D) disagree, and (E) strongly disagree.

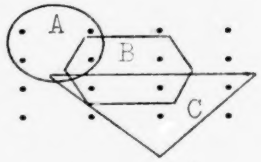
Internal consistency coefficients alpha for this scale were .69, .72, and .76 for the sixth grade, eighth grade, and high school subjects used in this study.

Achievement tests. The main achievement test was designed to test comprehension and application of the concepts in the instructional material. The achievement test was constructed from a large pool of items. Item data from prior instrument development studies were analyzed to arrive at the final version of the achievement test. It consisted of twenty-eight free response and seven multiple choice items (Appendix A). The first thirty items tested comprehension of the material presented in the probability text. For example, one item testing comprehension of the concepts of "probability," "event," and "outcome" is:

> If you toss three coins, P (at least one tail) = ________________.

The last five items required the student to use his comprehension of probability in a different format. For example, an item testing comprehension of the concepts of "event," "mutually exclusive," and "outcome" is:

Each of the 16 dots represents a possible outcome of an experiment. Assume the outcomes are equally likely.



A pair of events that is mutually exclusive is

(A) A,B (B) B,C (C) A,C (D) A,A
(E) None of these.

Internal consistency coefficients alpha calculated from experimental subjects data in the present study are reported in Table 2.

Table 2

Internal Consistency Coefficients Alpha
by Test Occasion*

		Test Session	
		Posttest	Retention Test
Subject Group	Sixth Grade	.902	.887
	Eighth Grade	.832	.827
	High School	.780	.794

*calculated from experimental Ss data

In addition to the thirty-five item achievement test, two ten item tests were given to the experimental subjects at the end of Sections 1 and 2 of the probability text, respectively (Appendix A). These tests only were used to give experimental Ss a progress check and to help insure that Ss did not proceed so quickly through the programmed material that little or no learning took place.

Word association test. The purpose of the word association (WA) test was to assess Ss' learning of mathematical structure. Ss received the following instructions for the WA test.

INSTRUCTIONS

This is a test to see how many words you can think of and write down in one minute.

You will be given a key word about probability and you are to write down all the words which the key word makes you think of.

Write down as many words as you can. You will probably not be able to fill in all of the spaces on a page, but do the best you can. Be sure to think of the key word after each word you write down, because the test is to see how many other words the key word makes you think of.

For example, suppose I asked a mathematician to write down as many words about mathematics as he could think of when given the word "set". He might put down the following

SET

Set element	Set union
Set number	Set__________

You will notice that as a mathematician he did not use "put" or "ready" since they are not words about mathematics.

In this same way, you should try to think of words about probability and mathematics that go with the key word.

You will have one minute on each page. I will tell you when to go to the next page.

Do you have any questions about what you are supposed to do?

The WA test consisted of one page of instructions and one page for each set of responses to each of the ten key concepts, respectively. On each response page, a key concept was printed at the top-center with the remainder of the page consisting of two columns of the key concept repeated with a blank to the right of the word. Four random sequences of the stimulus words (Appendix A) were used to prevent a possible sequence effect. A particular sequence was assigned randomly to S at each test administration.

Paragraph construction test. An alternate measure of the learning of structure was the paragraph construction (PC) test. Each PC test consisted of one page of instructions (reprinted below) and five pages for students' responses.

INSTRUCTIONS

This is a test to see how well you can explain how two words from probability are related. You will be given two words from probability. You are to write a paragraph which explains how the two words go together in probability. Write the paragraph as if you were explaining to a friend how the two words are related.

You may write as much as you need to explain how the two words are related in probability.

You will be given five pairs of words and you should explain how each pair is related. Each word-pair will be printed at the top of a page. You will have three minutes for each pair of words. I will tell you when to go to the next page.

For example, a student wrote about how WATER and STEAM are related. He explained the relation between WATER and STEAM as follows:

WATER --------STEAM

Water is a liquid made up of hydrogen and oxygen. When water is heated to its boiling point, it turns to a gas called steam. If steam is cooled, it turns back into water. Both water and steam are made of the same elements, hydrogen and oxygen.

Are there any questions?

You may turn the page and begin.

Each response page had a concept pair printed in the upper left-hand corner. The rest of the page was left blank for S to write a paragraph explaining the mathematical relationship between the two concepts listed at the top.

The number of Ss used in the study was not sufficiently large to allow the use of all possible (45) pairs of WA stimulus words since an excessive amount of testing time would be required. Since random sampling of pairs of words would not guarantee a representation of the variety of distances (determined by the digraph analysis) between concepts, pairs of concepts were chosen with the constraint that the set of pairs reflected the variation in distances between concepts. (This is a matrix sampling problem. See Lord and Novick, 1968, pp. 236-238.) Using the further constraints that S would be presented only five pairs of concepts and that the PC test would contain all ten stimulus words from the WA test, two versions of the PC test (see Table 3) were derived.

Table 3

The Two Versions of the Paragraph Construction Test

PC Test 1	PC Test 2
experiment--zero	zero--equally likely
equally likely--mutually exclusive	trial--independent
outcome--independent	outcome--mutually exclusive
probability--event	probability--experiment
trial--intersection	event--intersection

Four random orders (see Appendix A) of the concept pairs were used on each PC test. A particular order of concept pairs was assigned randomly to S as well as assigning either PC Test A or PC Test B at random to Ss.

Procedures

The study was conducted during regular school hours near the end of the 1971-1972 school year. Its duration varied from 21 to 29 calendar days. Orientation/pretesting/instruction/.../instruction/posttesting were carried out during consecutive class meetings. Several class days elapsed between the posttest and retention test sessions. Experimental Ss were never separated from control Ss during the experiment.

At least one proctor was available at each session to manage materials and procedures. Proctors did not instruct Ss, but answered procedural questions, read test instructions, etc. The regular teacher was present to maintain discipline.

The first class meeting in each subject group was devoted to orientation and pretesting. Prior to pretesting, Ss were told the experimentor was interested in finding out how students learn mathematics.

The attitude questionnaire, the WA test, and the achievement test on probability were administered, in the order listed, to all Ss prior to instruction. The attitude questionnaire was given first so that tests and treatments used in the experiment would not affect Ss' responses. The WA test was administered prior to the achievement test to insure that the achievement test did not acquaint Ss with possible responses to the WA test. It was felt that neither the attitude questionnaire nor the WA test would influence Ss' responses to the achievement test. A brief discussion of the purposes of the study (in lay terms) and how to use programmed instruction effectively followed the pretesting.

Each S then read the text assigned to him. At the end of each text section, S received a short review test over the section he had just completed. (The probability text did not have a test for Section 3, the final section.) Since instruction was self-paced not all Ss needed the entire instructional period to complete their reading of the text; conversely, not all Ss read the entire text. However, all Ss completed the second text section and most of the third section. Ss who finished early were allowed to read, draw, or study material of their choosing so long as the material was non-mathematical.

After instruction, all Ss were given the WA test, PC test, and achievement test, in the order listed. The WA test was given first so that the other instruments did not affect Ss' responses. It was reasoned that responses to the achievement test would be least affected by having Ss respond to the other instruments and thus it was given last.

Several days later the WA test and achievement test were readministered to Ss, in the sequence listed. The purpose of this test administration was to measure the subjects retention of the material.

Sixth and eighth grade Ss participated in their regular mathematics classes. High school Ss participated during their regular study hall or mathematics period. In the latter case, Ss were removed from their regular classroom.

Class periods were fifty, seventy-five, and fifty-three[7] minutes duration for sixth grade, eighth grade, and high school Ss respectively. The duration of testing and instruction were eleven, seven, and eight class meetings for grade six, eight, and high school subjects, respectively. The eighth grade classes met every other school day, all other classes met every school day. In some instances other school activities would cause a class not to meet on a particular school day. With the exception of one eighth grade class, all testing and instruction took place before noon.

Table 4 indicates the class meetings in which each subject group participated in the experiment, the days on

Table 4

Calendar Days on which Procedures were carried out

School Level	Pretest Orientation	Instruction	Posttest	Retention
Sixth Grade	1	2,3,4,5,8,9 10,11	11,12	29
Eighth Grade	1	3,7,9	11*	23
High School	1	3,4,8,9,10	10,11	21,22**

*Due to a conflicting school activity, one class of Ss was posttested on day 15.

**Due to school scheduling, half the Ss were retention-tested on day 21 while the remainder were tested on day 22.

---

[7] Due to other school activities, four periods were forty-two minutes in length.

which class meetings were held, and whether the meeting consisted of testing or instruction. The day-numbers represent calendar days.

Design

Following orientation and pretesting, Ss in three different school levels were divided at random into experimental and control groups. The experimental group received instruction in probability while the control group received instruction on an unrelated mathematical topic. After instruction, all Ss received a posttest and a retention test. Thus, in general, a treatment by grade by test occasion design was employed with repeated measures on the last factor. The tests used in the study and the test occasion are given in Table 5.

Table 5

Tests Administered and Test Occasions

Pretest	Posttest	Retention Test
Attitude Word Association Achievement	Word Association Paragraph Construction Achievement	Word Association Achievement

To summarize the design, achievement and word association testing was a repeated measure pre-post-retention design. The paragraph construction test was a post-only design. The attitude questionnaire was administered only at pretest.

CHAPTER IV

Results

This chapter contains the analyses of the data from the study. While the analyses are explained below and the results of each analysis are given, the interpretation of the results is found in Chapter V.

Content Structure

Digraph analysis. Content structure was mapped with the method of digraph analysis. The resulting digraph distance matrix is presented in Table 6. The greater the value of an element in the distance matrix, the greater the dissimilarity between the two concepts.

Table 6

Digraph Distance Matrix for Probability Text Structure

	Prob.	Ind.	Event	Zero	E.L.	Int.	Tr.	Exp.	M.E.	Out.
Probability	0	2	1	1	3	1	2	1	2	2
Independent	2	0	1	3	3	2	2	2	2	2
Event	1	1	0	2	2	1	1	2	1	1
Zero	1	3	2	0	4	2	3	2	3	3
Equally Likely	2	3	2	3	0	2	2	2	3	1
Intersection	1	2	1	2	2	0	2	2	2	1
Trial	3	3	2	4	2	2	0	1	2	1
Experiment	2	2	2	3	2	2	1	0	3	1
Mutually Exclusive	2	2	1	2	3	2	2	3	0	2
Outcome	2	2	1	3	1	1	1	1	2	0

The values in the digraph distance matrix represent dissimilarities between concepts. The values in the relatedness coefficient matrix, a representation of cognitive structure, represent similarities between pairs of concepts. In order to compare these two types of matrices, the digraph distance matrix was converted into a similarity matrix (see Table 7). Shavelson (1970) performed this modification by interchanging the largest element and the smallest element, the next largest element and the next smallest element, etc. and then dividing each element in the distance matrix by the largest element in the matrix. However, this method results in a division performed solely on the basis of a particular matrix. A more general technique, namely dividing one by the element plus one, is preferred. (Thus the element 2 would be replaced by 1/(2+1) or .333, the element 1 would be replaced by .5, etc.) This method has the advantage of making comparisons between studies possible. Also, should an element be infinite, Shavelson's procedure would result in an entire matrix of zeros. The more general method results in zeros only for the infinite elements. Note that the digraph similarity matrix will contain only numbers between zero and one, inclusive, and thus "resemble" the relatedness coefficient matrix.

Table 7

Digraph Similarity Matrix for Probability Text Structure

	Prb.	Ind.	Event	Zero	E.L.	Int.	Tr.	Exp.	M.E.	Out.
Probability	1	.33	.5	.5	.25	.5	.33	.5	.33	.33
Independent	.33	1	.5	.25	.25	.33	.33	.33	.33	.33
Event	.5	.5	1	.33	.33	.5	.5	.33	.5	.5
Zero	.5	.25	.33	1	.20	.33	.25	.33	.25	.25
Equally Likely	.33	.25	.33	.25	1	.33	.33	.33	.25	.5
Intersection	.5	.33	.5	.33	.33	1	.33	.33	.33	.5
Trial	.25	.25	.33	.20	.33	.33	1	.5	.33	.5
Experiment	.33	.33	.33	.25	.33	.33	.5	1	.25	.5
Mutually Exclusive	.33	.33	.5	.33	.25	.33	.33	.25	1	.33
Outcome	.33	.33	.5	.25	.5	.5	.5	.5	.33	1

From a matrix of similarities between concepts, it is helpful to obtain a measure of the organization of the concepts, i.e. how they fit into a given space. Shuell (1969) discusses clustering in free recall. He develops various indices to measure clustering and discusses theoretical issues concerning clustering. It appears his approach might be useful for determining pre-existing structure in memory, but it depends on having S learn word lists to obtain a baseline and thus is not applicable to the present study. Thomas Johnson (1969) gives definitions and reviews literature concerning cognitive structure and the use of proximity measures. Of special interest is his discussion concerning a unique metric solution arrived at from data which is essentially non-metric.

Kruskal (1964) discusses multidimensional scaling and gives a stronger mathematical foundation for Shepard's technique. Since multidimensional scaling assumes a dimensionality to the structure and this is consistent with our understanding of mathematics it was selected for the present study. This technique requires no metric assumptions and yields a best-fitting geometric representation in a space of the smallest number of dimensions. Kruskal (p. 3) gives the following guidelines in determing how well the data fit in a given space:

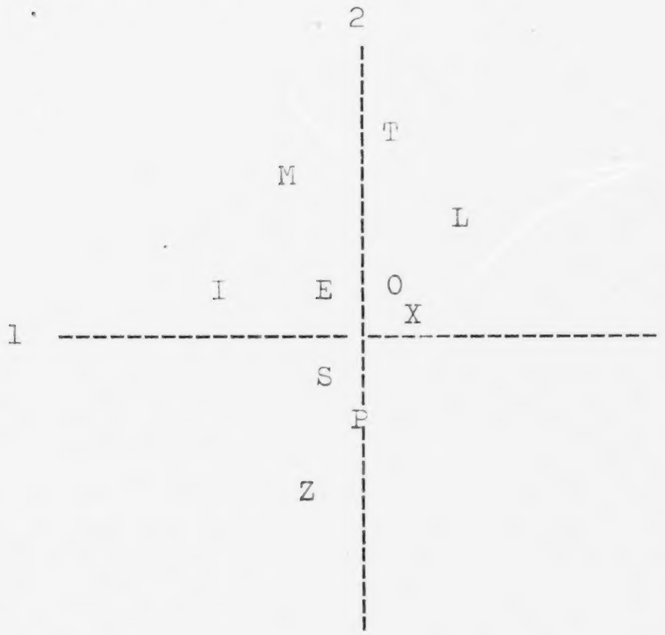
Stress	Goodness of Fit
.200	poor
.100	fair
.050	good
.025	excellent
.000	perfect

The interrelations among concepts in the digraph similarity matrix were examined with Kruskal's (1964) multidimensional scaling procedure. For the purposes of this study, it was decided to accept the smallest number of dimensions that would allow a "good" fit to the data. Figure 3 presents the graphical representation of the results. Appendix B contains the numerical results of the analysis.

Graph analysis. Graph analysis also was used to examine content structure presented by the probability text. As noted in Chapter III, the essential difference between graph analysis and digraph analysis is that graph analysis does not use directed lines. That is, the graph analysis results in a symmetric distance matrix (see Table 8).

Figure 3

Plot of Multidimensional Scaling Solution
Content Digraph Analysis



Key for Plots

P = Probability	S = Intersection
I = Independent	T = Trial
E = Event	X = Experiment
Z = Zero	M = Mutually Exclusive
L = Equally Likely	O = Outcome

Table 8

Graph Distance Matrix for Probability
Text Structure

	Prb.	Ind.	Event	Zero	E.L.	Int.	Tr.	Exp.	M.E.	Out.
Probability	0	2	1	1	2	1	2	1	2	2
Independent	2	0	1	3	3	2	2	2	2	2
Event	1	1	0	2	2	1	1	2	1	1
Zero	1	3	2	0	3	2	3	2	2	3
Equally Likely	2	3	2	3	0	2	2	2	3	1
Intersection	1	2	1	2	2	0	2	2	2	1
Trial	2	2	1	3	2	2	0	1	2	1
Experiment	1	2	2	2	2	2	1	0	3	1
Mutually Exclusive	2	2	1	2	3	2	2	3	0	2
Outcome	2	2	1	3	1	1	1	1	2	0

Again it is helpful to convert the graph distance matrix into a similarity matrix using the same procedure as was used for the digraph distance matrix (See Table 9). Kruskal's (1964) multidimensional scaling was performed on the graph similarity matrix and the plot of the results are shown in Figure 4. Appendix B contains the numerical results of the multidimensional scaling procedure.

Task analysis. Using procedures suggested by Gagne (1962, 1965, 1970), a task analysis was performed on the probability text to examine the heirarchy of completeness represented in the subject matter. The resultant heirarchy is presented in Figure 5. The investigator was not able to determine a satisfactory method for obtaining a "distance"

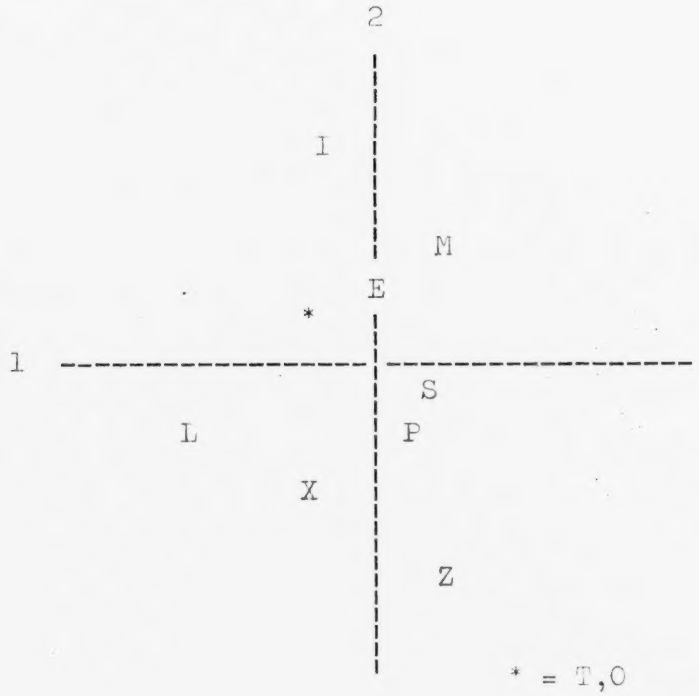
Table 9

Graph Similarity Matrix for Probability Text Structure

	Prb.	Ind.	Event	Zero	E.L.	Int.	Tr.	Exp.	M.E.	Out.
Probability	1	.33	.5	.5	.33	.5	.33	.5	.33	.33
Independent	.33	1	.5	.25	.25	.33	.33	.33	.33	.33
Event	.5	.5	1	.33	.33	.5	.5	.33	.5	.5
Zero	.5	.25	.33	1	.25	.33	.25	.33	.33	.25
Equally Likely	.33	.25	.33	.25	1	.33	.33	.33	.25	.5
Intersection	.5	.33	.5	.33	.33	1	.33	.33	.33	.5
Trial	.33	.33	.5	.25	.33	.33	1	.5	.33	.5
Experiment	.5	.33	.33	.33	.33	.33	.5	1	.25	.5
Mutually Exclusive	.33	.33	.5	.33	.25	.33	.33	.25	1	.33
Outcome	.33	.33	.5	.25	.5	.5	.5	.5	.33	1

Figure 4

Plot of Multidimensional Scaling Solution
Content Graph Analysis



Key for Plots

P = Probability
I = Independent
E = Event
Z = Zero
L = Equally Likely

S = Intersection
T = Trial
X = Experiment
M = Mutually Exclusive
O = Outcome

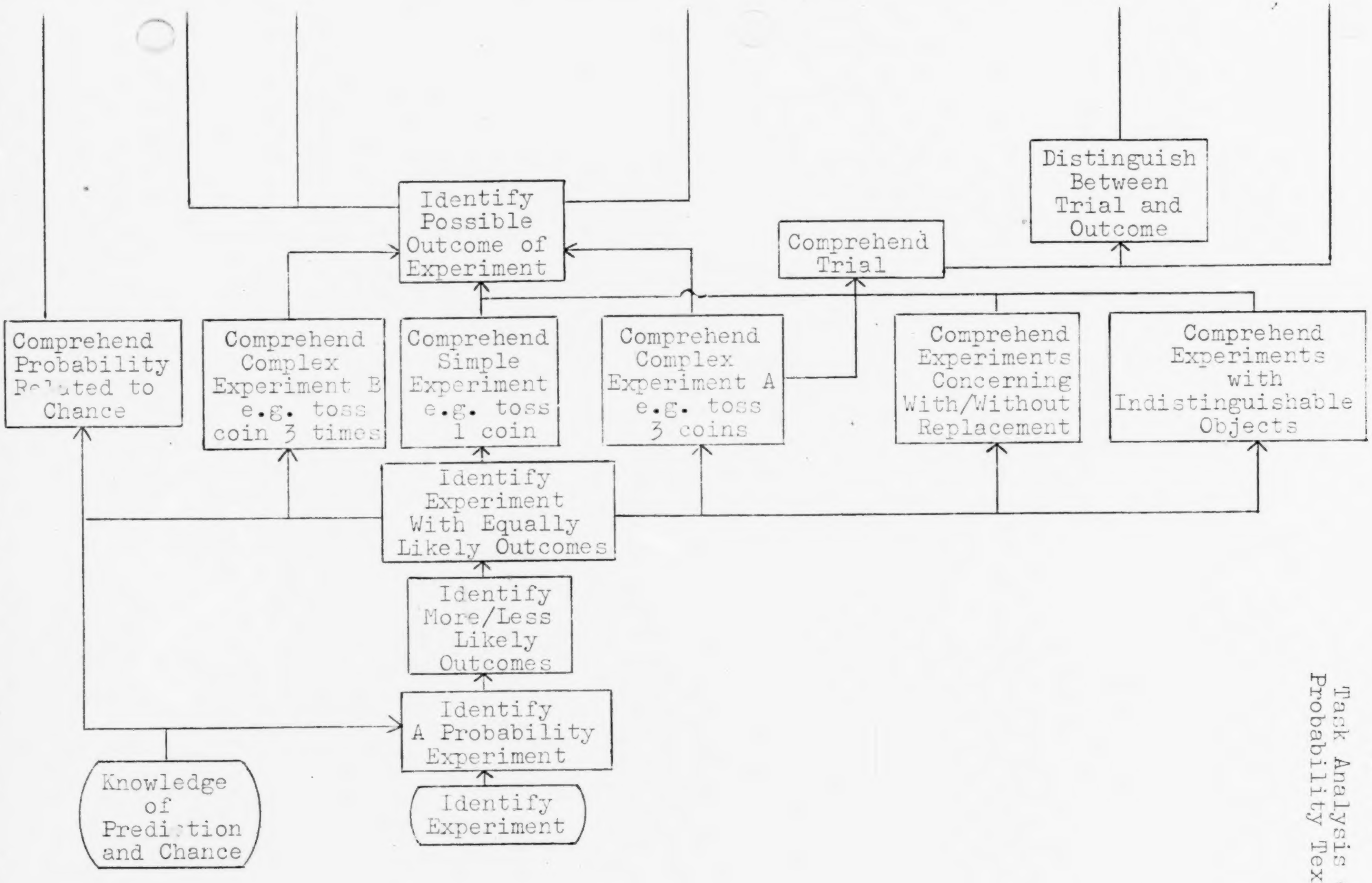

Distinguish Between Trial and Outcome
Identify Possible Outcome of Experiment
Comprehend Trial
Comprehend Probability Related to Chance
Comprehend Complex Experiment B e.g. toss coin 3 times
Comprehend Simple Experiment e.g. toss 1 coin
Comprehend Complex Experiment A e.g. toss 3 coins
Comprehend Experiments Concerning With/Without Replacement
Comprehend Experiments with Indistinguishable Objects
Identify Experiment With Equally Likely Outcomes
Identify More/Less Likely Outcomes
Identify A Probability Experiment
Knowledge of Prediction and Chance
Identify Experiment

Figure 5

Task Analysis of Probability Text*

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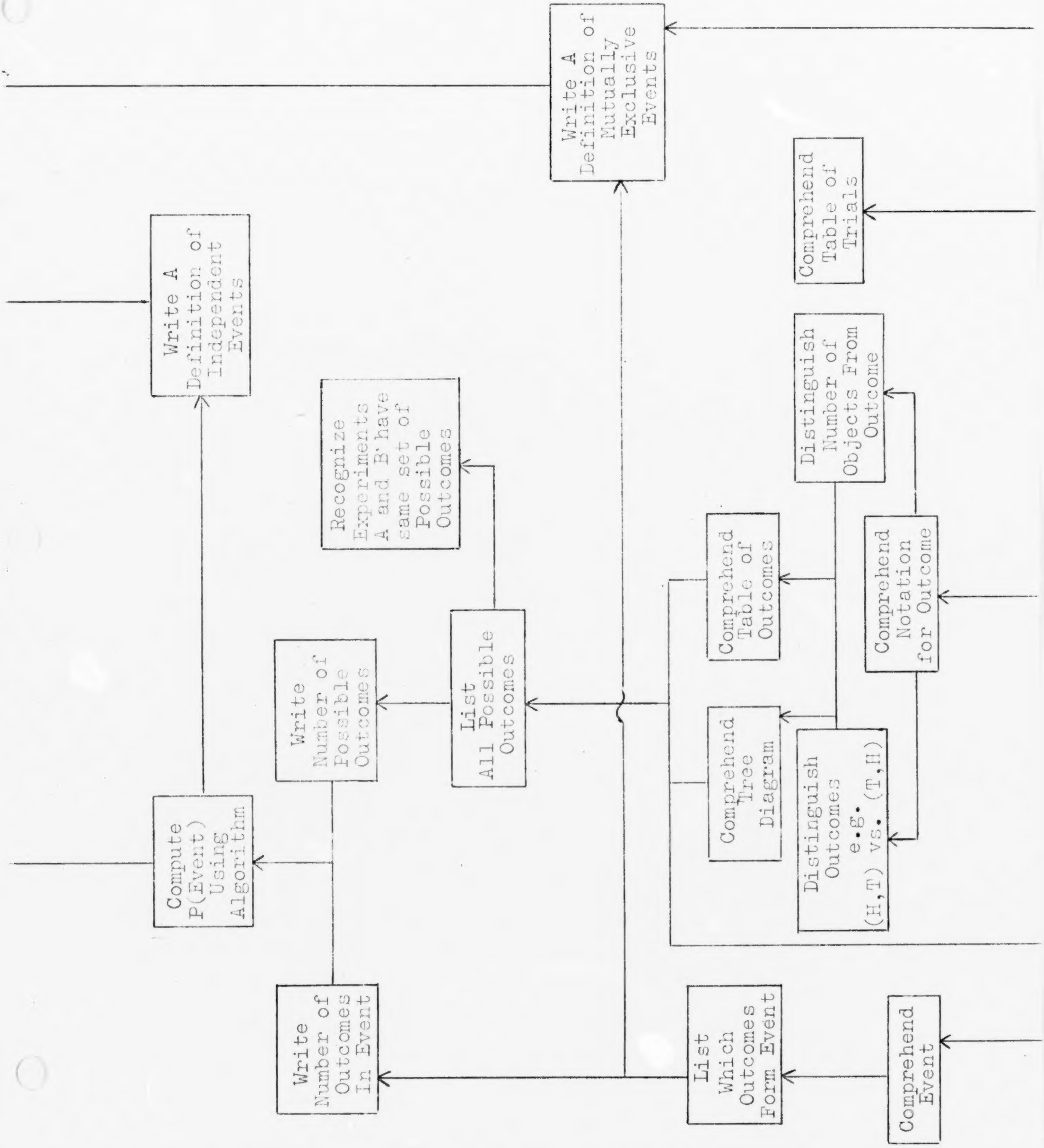
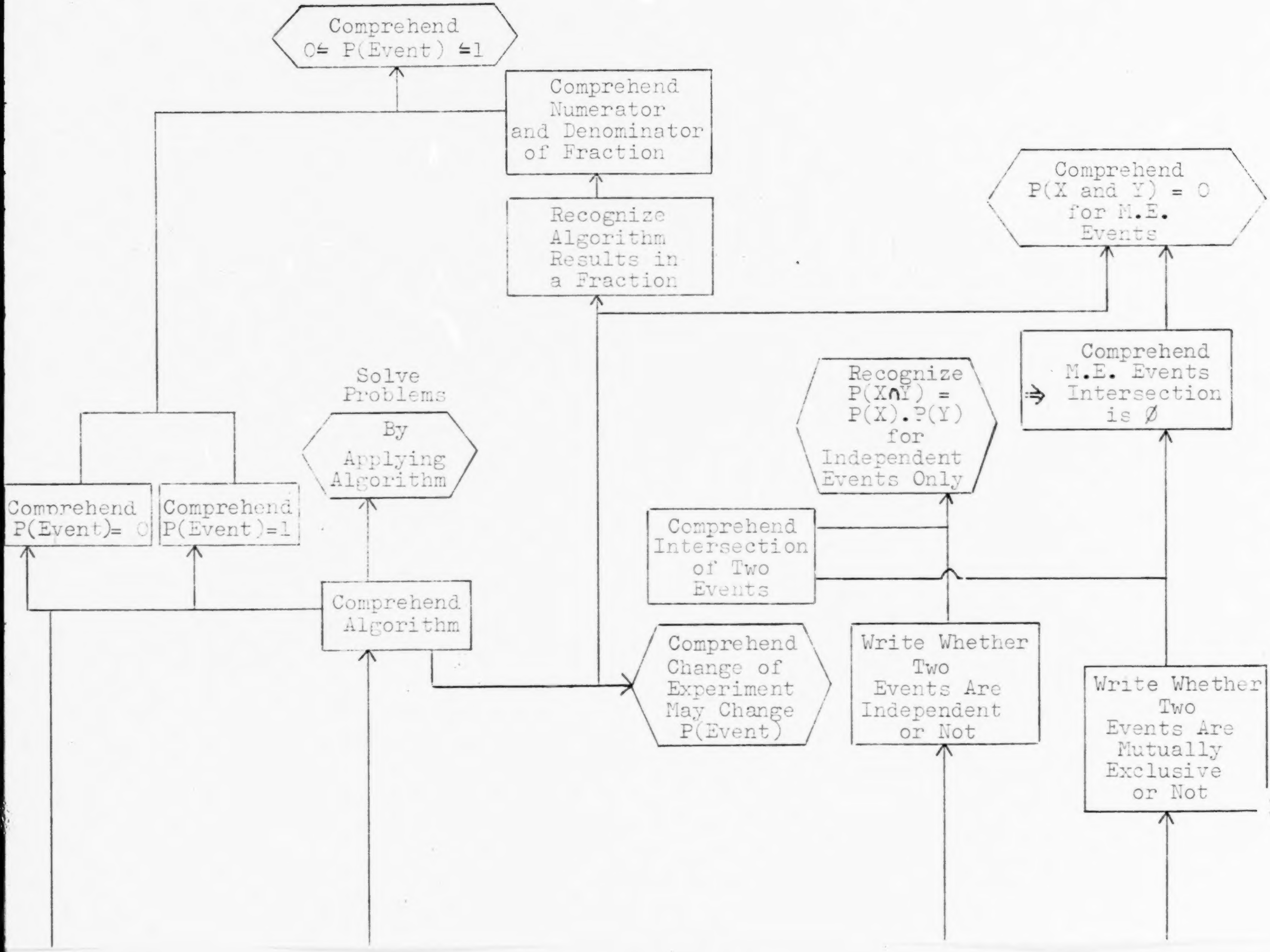
*The use of the word "comprehend" in the task analysis deviates somewhat from Gagne's (1970) terminology. A definition of the word comprehend and a discussion of its use is given by Romberg and Wilson (1969).


Write A Definition of Mutually Exclusive Events
Write A Definition of Independent Events
Comprehend Table of Trials
Distinguish Number of Objects From Outcome
Recognize Experiments A and B have same set of Possible Outcomes
Comprehend Table of Outcomes
Comprehend Notation for Outcome
Write Number of Possible Outcomes
List All Possible Outcomes
Comprehend Tree Diagram
Distinguish Outcomes e.g. (H,T) vs. (T,H)
Compute P(Event) Using Algorithm
Write Number of Outcomes In Event
List Which Outcomes Form Event
Comprehend Event


Comprehend
0≤ P(Event) ≤1
Comprehend
Numerator
and Denominator
of Fraction
Recognize
Algorithm
Results in
a Fraction
Comprehend
P(X and Y) = 0
for M.E.
Events
Comprehend
M.E. Events
⇒ Intersection
is Ø
Recognize
P(X∩Y) =
P(X).P(Y)
for
Independent
Events Only
Solve
Problems
By
Applying
Algorithm
Comprehend
P(Event)= 0
Comprehend
P(Event)=1
Comprehend
Intersection
of Two
Events
Comprehend
Algorithm
Comprehend
Change of
Experiment
May Change
P(Event)
Write Whether
Two
Events Are
Independent
or Not
Write Whether
Two
Events Are
Mutually
Exclusive
or Not

matrix from the resultant heirarchy. One could count "boxes" between concepts, but the boxes do not represent concepts alone but rather they represent manipulations or performances with concepts. Thus, for example, the concept "outcome" and "event" appear in several boxes and one could arrive at several distances between these concepts depending on the boxes selected. Additionally, the boxes are derived in a somewhat subjective manner. A logical analysis by one author may not be the same as a logical analysis for a second author; thus causing the two authors to arrive at different distance matrices. The task analysis should be useful in interpreting the other content-analyses and the analyses of the WA data, but does not appear to be a satisfactory representation of structure as we have defined it.

Achievement, Ability, and Attitude Data

Sixth grade subjects. Lorge-Thorndike (L/T) verbal, non-verbal, and total scores as well as the California Test of Basic Skills (CTBS) reading and arithmetic scores were available for the sixth grade Ss. Descriptive statistics for these data are given in Table 10. Table 10 also contains descriptive statistics for the "Attitude Toward Mathematics" and pretest, posttest, and retention test achievement data.

Achievement data were analyzed by a 2 x 3 (treatment by test occasion) analysis of variance with repeated measures on the second factor. Results indicated the treatment effect was significant ($F = 22.62$, $df = 1/40$, $p < .01$) with experimental Ss scoring higher than control Ss (see Table 10). The

Table 10

Descriptive Statistics for Sixth Grade Subjects
Ability, Attitude, and Achievement Data

		Lorge-Thorndike VERB.	Lorge-Thorndike NONV.	Lorge-Thorndike TOTAL	CTBS READ	CTBS ARITH.	ATTITUDE TO MATH.	ACHIEVEMENT PRE	ACHIEVEMENT POST	ACHIEVEMENT RET
Exp.	Sample Size	30	30	30	30	30	32	31	31	23
	Mean	97.400	107.600	102.767	5.837	5.690	32.406	1.258	10.452	9.783
	Standard Deviation	16.089	17.132	15.808	1.742	1.558	5.858	1.316	7.270	7.292
	Range Max	141	144	143	10.0	9.3	46	5	26	24
	Range Min	63	73	68	2.8	3.0	23	0	0	1
Con.	Sample Size	21	21	21	22	22	25	25	23	22
	Mean	96.000	104.143	100.333	5.623	5.632	30.720	1.280	1.565	2.000
	Standard Deviation	11.857	15.723	12.403	1.647	1.418	7.956	1.173	2.041	2.430
	Range Max	120	131	121	8.7	8.7	45	4	9	8
	Range Min	73	70	75	2.6	3.2	16	0	0	0

test occasion effect was significant ($F = 18.95$, df = 2/39, $p < .01$); and the interaction effect (treatment x test occasion) was significant ($F = 14.16$, df = 2/39, $p < .01$).

Since there was a loss of several grade six subjects at retention test, a second analysis was performed. This was a 2 x 2 (treatment by test occasion) analysis of variance with repeated measures on the second factor (test occasions were pretest and posttest). The results obtained were: a) a significant treatment effect ($F = 25.69$, df = 1/51, $p < .01$); b) a significant test occasion effect ($F = 40.17$, df = 1/51, $p < .01$); and c) a significant interaction between treatment and test occasion effects ($F = 34.31$, df = 1/51, $p < .01$).

Eighth grade subjects. Scores on the Minimum Essentials of Modern Mathematics (MEMM) test were available for eighth grade Ss. Table 11 describes these data as well as the attitude and achievement data.

Achievement data were analyzed by a 2 x 3 (treatment by test occasion) analysis of variance with repeated measures on the second factor. Results obtained were: a) a significant treatment effect ($F = 114.92$, df = 1/76, $p < .01$), experimental Ss scored higher than control Ss (see Table 11); b) a significant test occasion effect ($F = 86.85$, df = 2/75, $p < .01$); and c) a significant interaction between effects ($F = 63.55$, df = 2/75, $p < .01$).

High school subjects. No ability data were available for high school Ss. Descriptive statistics concerning the data obtained from the attitude and achievement tests are presented in Table 12.

Table 11

Descriptive Statistics for Eighth Grade Subjects Ability, Attitude, and Achievement Data

				ACHIEVEMENT		
		MEMM[*]	ATTITUDE TO MATH	PRETEST	POSTTEST	RETENTION
Exp.	Sample Size	27	43	43	41	43
	Mean	55.259	30.302	3.651	15.537	16.209
	Standard Deviation	20.350	5.998	2.448	5.736	6.315
	Range Max	82	45	8	27	31
	Range Min	11	15	0	3	1
Con.	Sample Size	24	42	42	40	43
	Mean	60.396	28.833	3.000	3.725	4.163
	Standard Deviation	25.569	5.587	1.900	2.460	3.062
	Range Max	99.5	40	9	8	11
	Range Min	11	19	0	0	0

[*] Minimum Essentials of Modern Mathematics

Table 12

Descriptive Statistics for High School
Subjects Attitude and Achievement Data

		ATTITUDE TO MATH	ACHIEVEMENT PRETEST	POSTTEST	RETENTION
Exp.	Sample Size	20	19	15	11
	Mean	32.300	4.526	21.000	24.636
	Standard Deviation	6.157	3.502	5.014	5.259
	Range Max	44	14	28	33
	Range Min	21	0	12	18
Con.	Sample Size	14	14	11	6
	Mean	31.714	5.500	6.455	7.833
	Standard Deviation	7.559	4.832	3.934	5.636
	Range Max	41	19	14	17
	Range Min	17	0	1	2

A 2 x 3 (treatment by test occasion) analysis of variance with repeated measures on the second factor was used to analyze the achievement data. A significant treatment effect (F = 22.55, df = 1/14, $p < .01$), a significant test occasion effect (F = 25.19, df = 2/13, $p < .01$), and a significant interaction between effects (F = 24.59, df = 2/13, $p < .01$) were found.

Since there was a loss of several subjects at retention test time, a 2 x 2 (treatment by test occasion) analysis of variance with repeated measures on the second factor also was used to analyze the achievement data (pretest and posttest data only). Results were a significant treatment effect ($F = 25.65$, $df = 1/24$, $p < .01$), a significant test occasion effect ($F = 90.05$, $df = 1/24$, $p < .01$), and a significant interaction term ($F = 72.32$, $df = 1/24$, $p < .01$).

Comparisons among school levels. To investigate the effect of school level, a 2 x 3 x 2 (treatment by school level by test occasion) analysis of variance with repeated measures on the third factor was performed. The retention test data were not included in this analysis due to loss of Ss in the sixth grade and high school levels at retention test. Of interest here was a significant school level effect ($F = 28.08$, $df = 2/152$, $p < .01$), a significant treatment effect ($F = 117.34$, $df = 1/152$, $p < .01$), a significant test occasion effect ($F = 86.81$, $df = 1/152$, $p < .01$), and a non-significant interaction between treatment and school level effects ($F = 2.06$, $df = 2/152$).

Cognitive Structure

Cognitive structure was investigated with word association (WA) and paragraph construction (PC) techniques. Results were analyzed in a 2 x 3 x 3 (treatment group by school level by test occasion) design. Each set of WA data yields a symmetric relatedness coefficient (RC) matrix representing the relationship between pairs of key concepts in memory. Since these matrices are similarity matrices they may be compared to the similarity matrices obtained in the analyses of content structure. The structure represented by each RC matrix was examined by Kruskal's (1964) multidimensional scaling procedure.

Analysis of WA responses. Marshall and Cofer (1963) review ten indices which convert WA data to a numerical index indicating the degree of relatedness between concepts. Many of these indices are not applicable to the present study since they handle only two concepts or else deal with WA techniques that allow only one response to each stimulus concept.

The method selected in this study to convert WA data into a matrix of similarities between concepts is the relatedness coefficient proposed by Garskof and Houston (1963). The relatedness coefficient (RC) depends on the number of responses to a given stimulus word and the overlap between response distributions for pairs of stimulus words. The formula for obtaining the RC coefficient is:

$$RC = \frac{\overline{A} \cdot \overline{B}}{(A \cdot B) - [n^{P} - (n-1)^{P}]^{2}}$$

where

- $\overline{A}$ and $\overline{B}$ represent the rank order of words under A which are shared in common with B and the rank order of words in B which are shared in A.
- $\overline{A} \cdot \overline{B}$ represents the rank order of words in A multiplied by the rank order of words in B.
- n represents all of the words in B (the longer list).
- P represents some fixed number greater than zero which may be determined from the shape

> of the probability distribution of the responses. P was set equal to 1 in this study; all portions of the S's response distribution received equal weight.

The RC coefficient may have a ceiling effect as suggested by Shavelson (1970). Additionally the RC coefficient is symmetric and thus would not be able to reproduce a digraph distance matrix (asymetric) exactly. Garskof and Houston (1963) examine the validity of the RC coefficient.

Each relatedness coefficient may range from zero to one inclusive and indicates the degree to which two concepts are related to S's memory. The larger the value of the relatedness coefficient the closer the relationship between the two concepts. For example, an eighth grade S responded to event and experiment on the post WA test as follows:

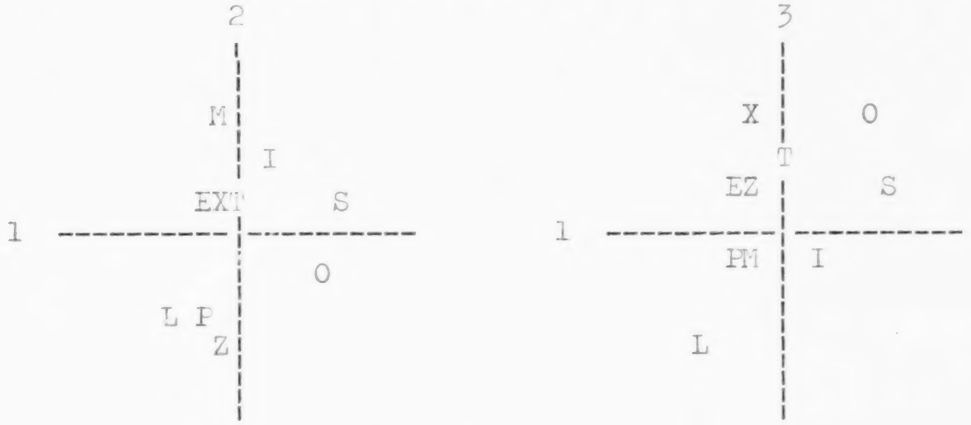
Event	Rank	Experiment	Rank
Event	5	Experiment	5
Number	4	Event	4
Trial	3	Outcome	3
Outcome	2	Trial	2
		Probability	-

$$\text{Thus}\quad RC = \frac{(5\ \ 3\ \ 2)\cdot\begin{pmatrix}4\\2\\3\end{pmatrix}}{(5\ \ 4\ \ 3\ \ 2\ \ 1)\cdot\begin{pmatrix}5\\4\\3\\2\\1\end{pmatrix} - [5^1-(5-1)^1]^2} = .593$$

For each S a 10 x 10 RC matrix was formed using the relatedness coefficients obtained from that S's WA responses. For each cell of the 2 x 3 x 3 design (treatment group by school level by test occasion) a mean RC matrix and median RC matrix (calculated element by element) were formed. Shavelson (1970) used only a median RC matrix. The underlying normality (or lack of it) of relatedness coefficients may be argued, but the author prefers to present both the mean and median RC matrices. The RC matrices obtained from the WA data may be found in Appendix C. Each RC matrix was scaled using Kruskal's (1964) procedure; the results may be found in Figures 6-30. When most off-diagonal elements were zero for a particular RC matrix, e.g. pretest WA data, no scaling solution is given as the procedure is not applicable to such a matrix. Appendix B contains the numerical results of the scaling solutions.

Figure 6**

Plot of Multidimensional Scaling Solution
Sixth Grade Experimental Subjects Pretest
Mean Relatedness Coefficient Matrix



**Key for Figures 6-35.

P = Probability
I = Independent
E = Event
Z = Zero
L = Equally Likely
S = Intersection
T = Trial
X = Experiment
M = Mutually Exclusive
O = Outcome

*,#indicates more than one concept located at a particular point.

Figure 6 (cont.)

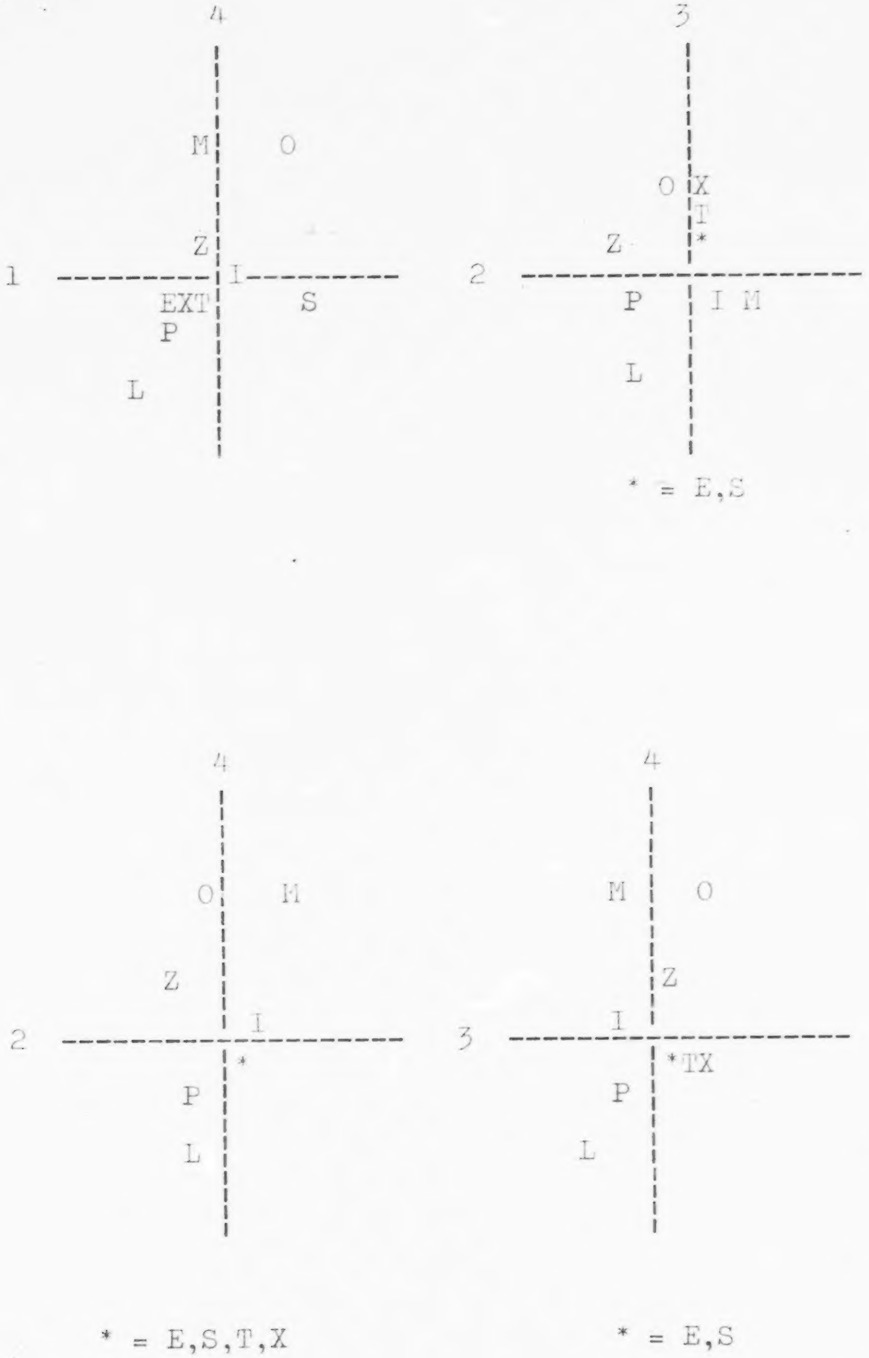


Figure 7

Plot of Multidimensional Scaling Solution
Sixth Grade Experimental Subjects Posttest
Mean Relatedness Coefficient Matrix

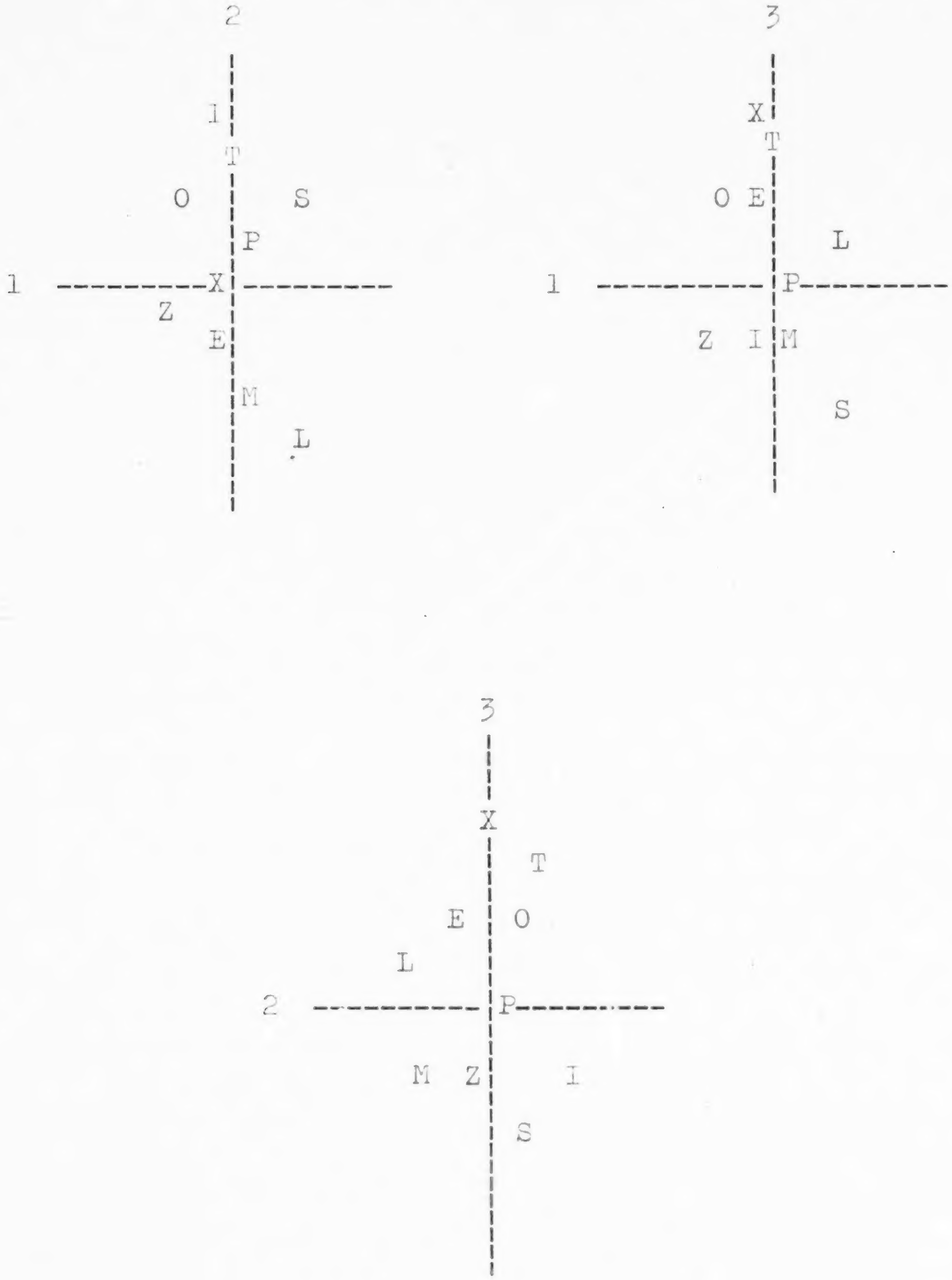


Figure 8

Plot of Multidimensional Scaling Solution
Sixth Grade Experimental Subjects Posttest
Median Relatedness Coefficient Matrix

```
                2
                |
                |
                |
                |
                |
1  ---O-----E---*--S---------P
                |
                |
                |
                |
                |
                |

                      * = I,Z,L,T,X,M
```

Figure 9

Plot of Multidimensional Scaling Solution
Sixth Grade Experimental Subjects Retention
Mean Relatedness Coefficient Matrix

```
              2
              |
              |
            I |
              |
              |X
        L   M |T
1  ---------P-| -------------
             E|
             O|      S
              |
              | Z
              |
              |
```

Figure 9 (cont.)

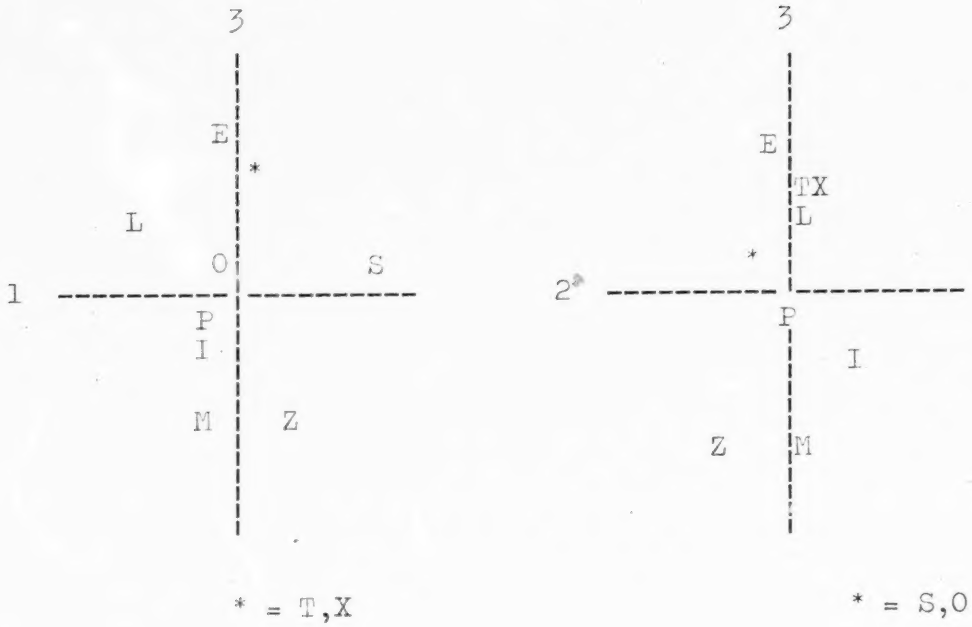


Figure 10

Plot of Multidimensional Scaling Solution
Sixth Grade Experimental Subjects Retention
Test Median Relatedness Coefficient Matrix

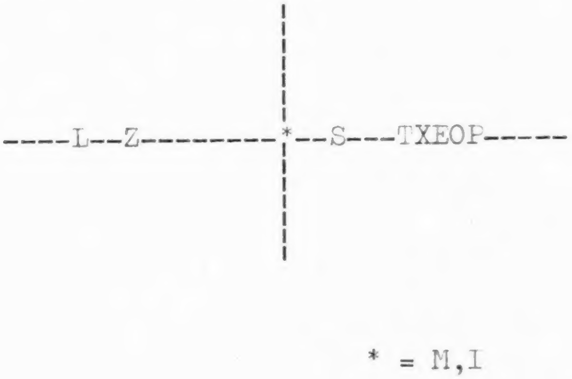
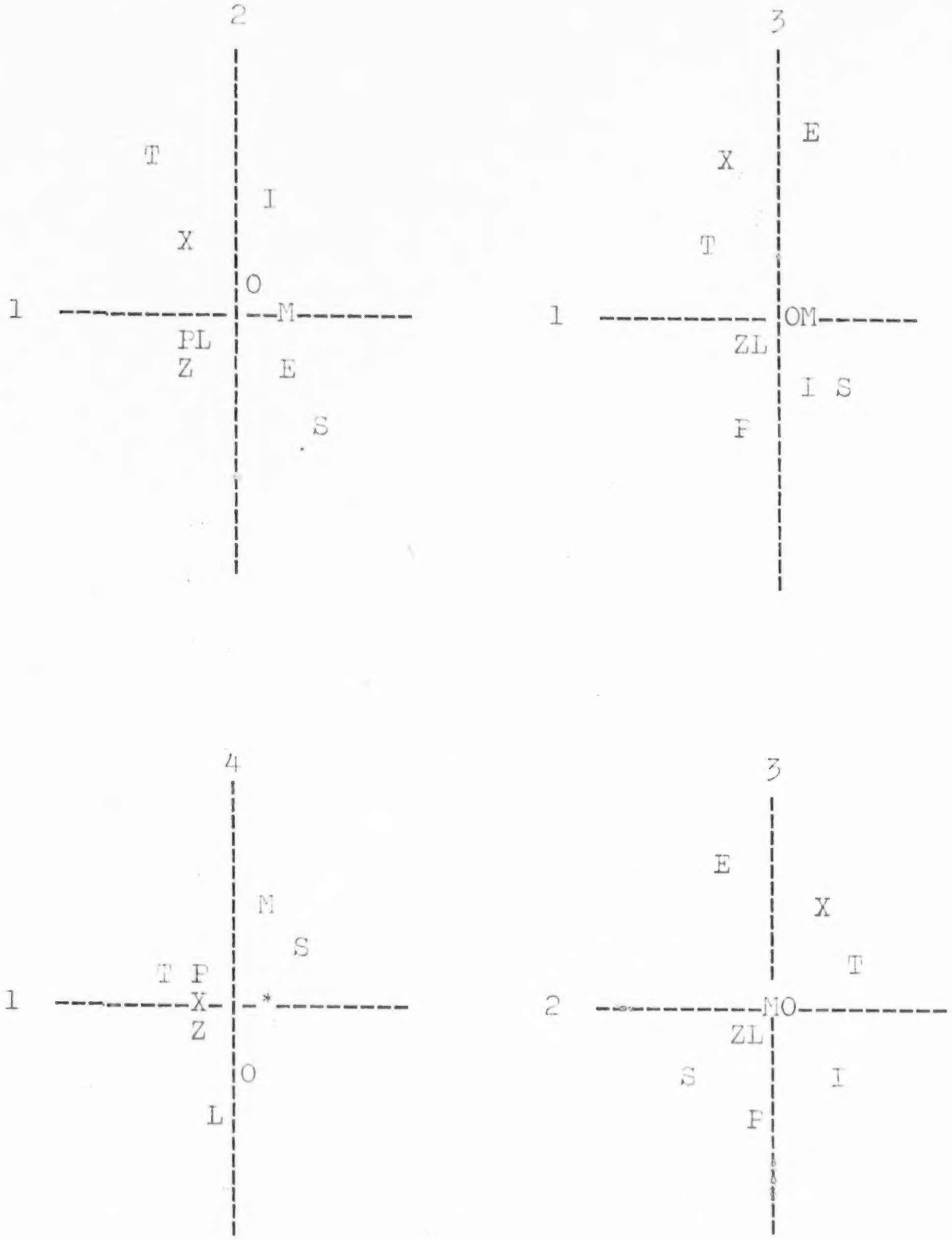


Figure 11

Plot of Multidimensional Scaling Solution
Sixth Grade Control Subjects Pretest Mean
Relatedness Coefficient Matrix



* = I,E

Figure 11 (cont.)

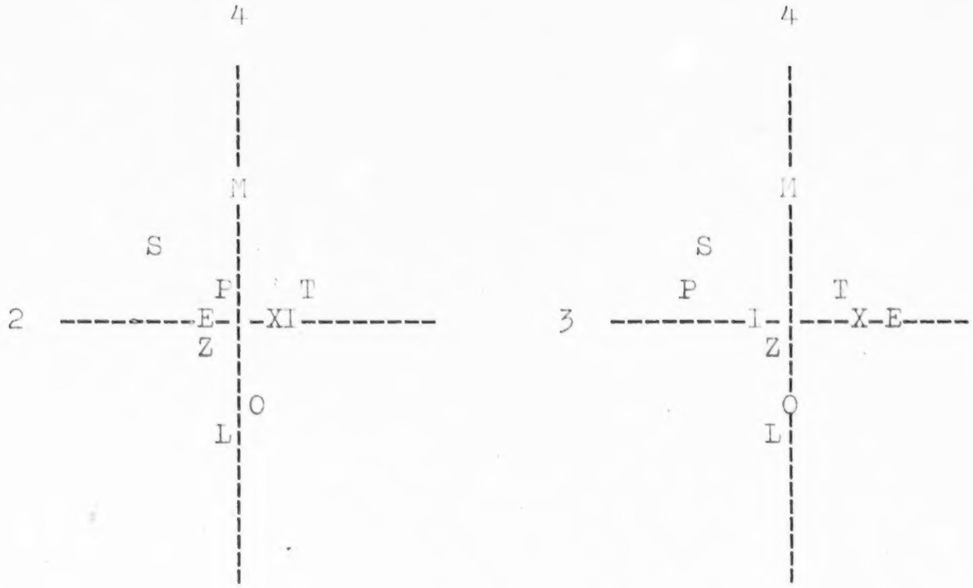

4
M
S
P
T
2
E
XI
Z
O
L
4
M
S
P
T
3
I
X
E
Z
O
L

Figure 12

Plot of Multidimensional Scaling Solution Sixth Grade Control Subjects Posttest Mean Relatedness Coefficient Matrix

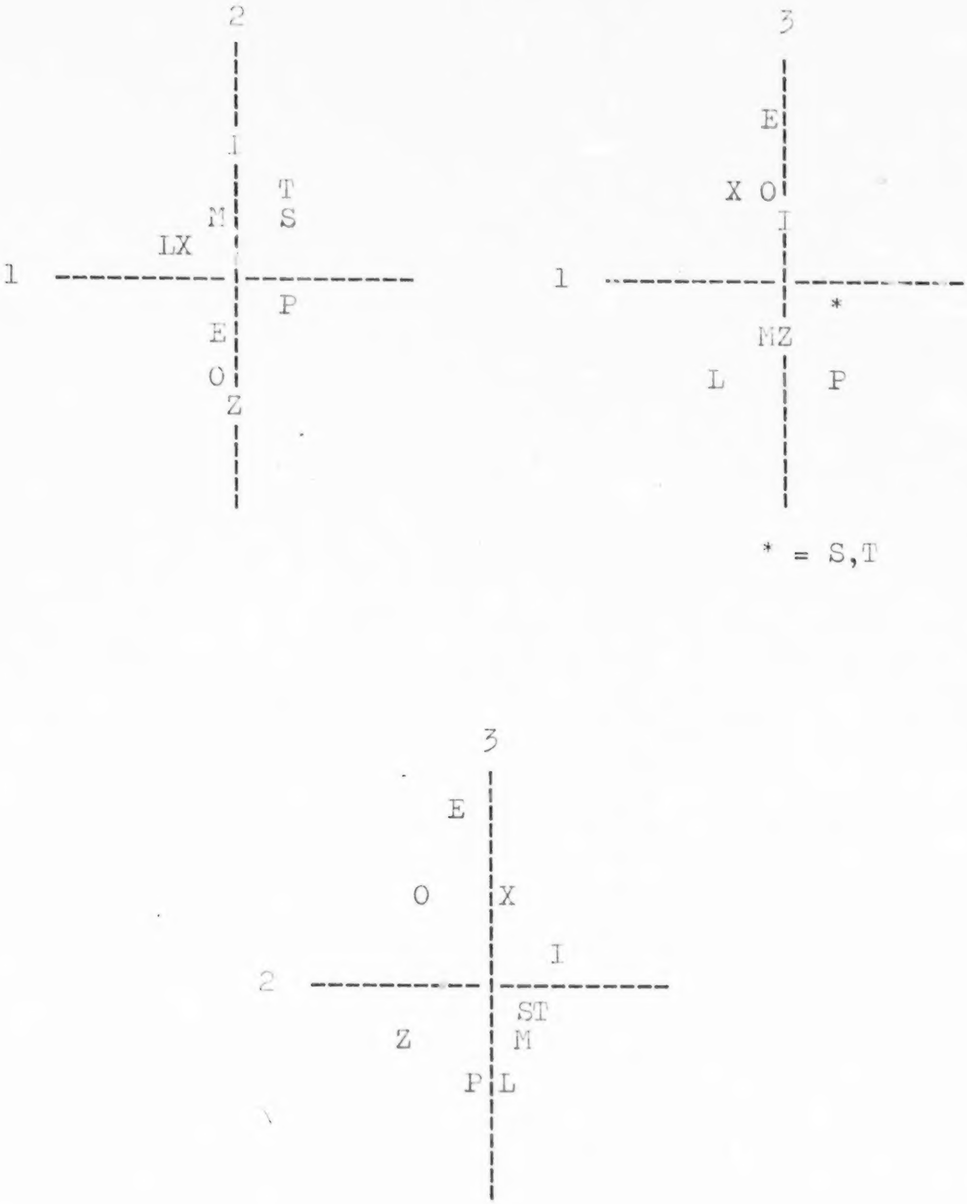


Figure 13

Plot of Multidimensional Scaling Solution
Sixth Grade Control Subjects Retention Test
Mean Relatedness Coefficient Matrix

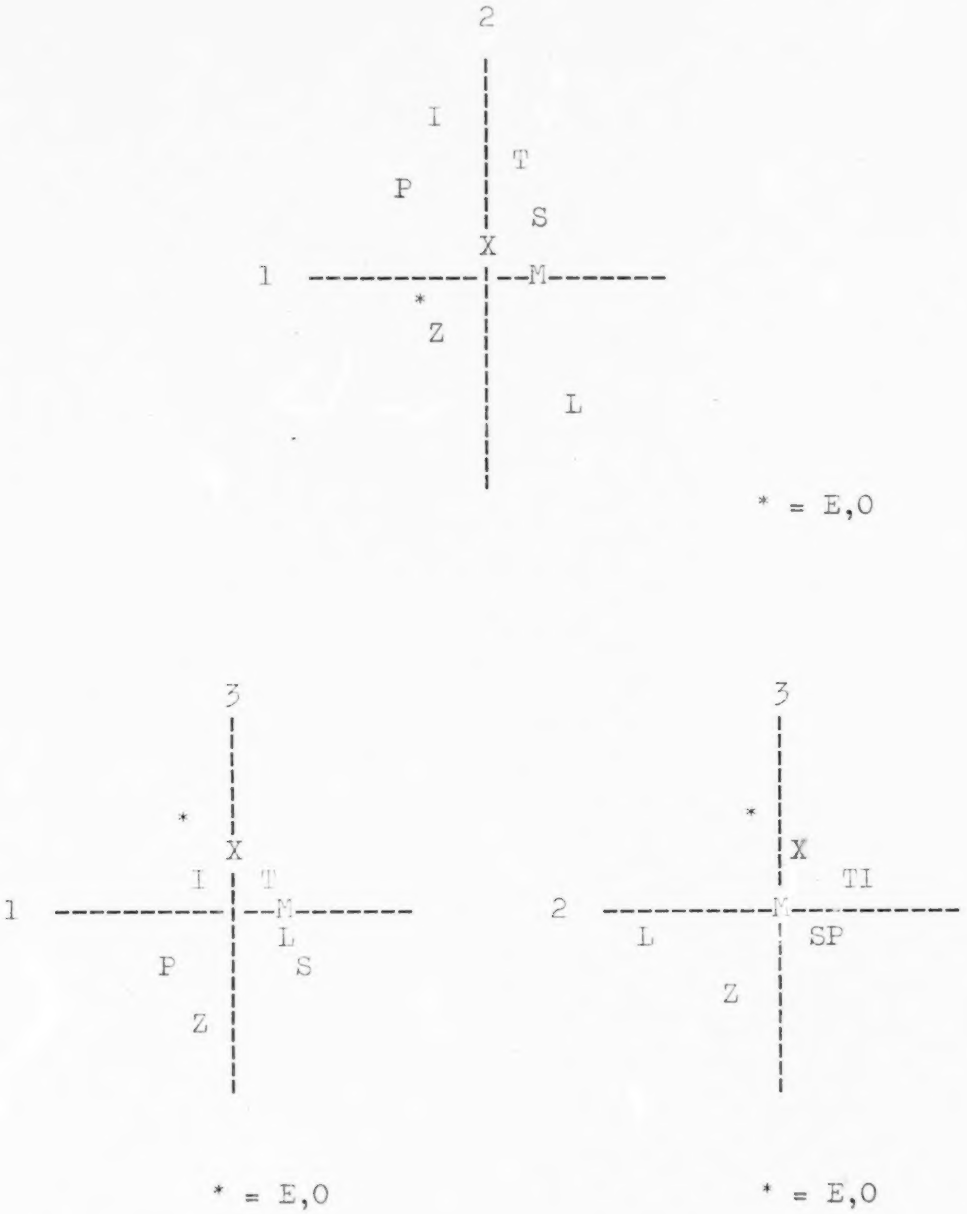


Figure 14

Plot of Multidimensional Scaling Solution
Eighth Grade Experimental Subjects Pretest
Mean Relatedness Coefficient Matrix

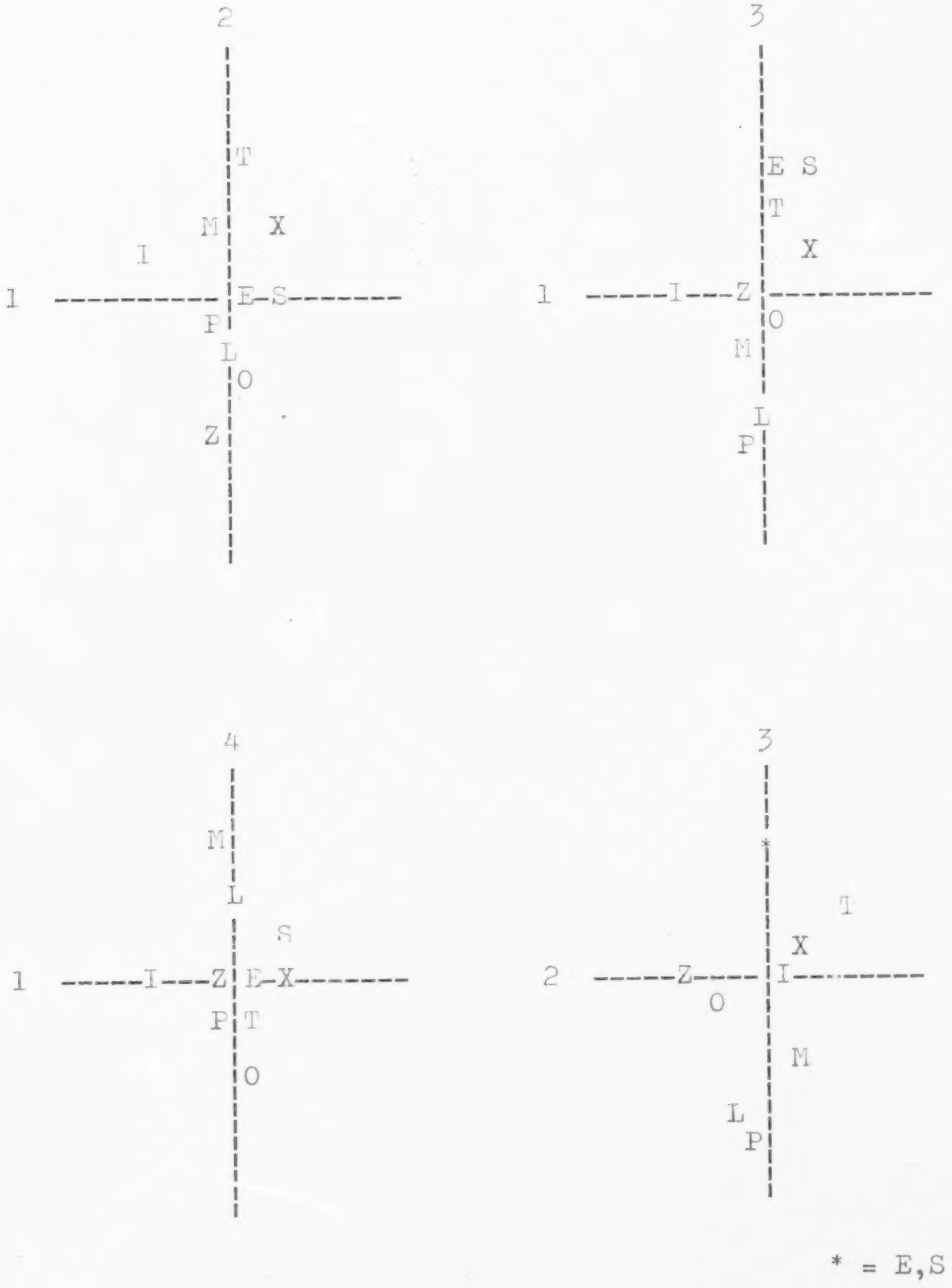


Figure 14 (cont.)

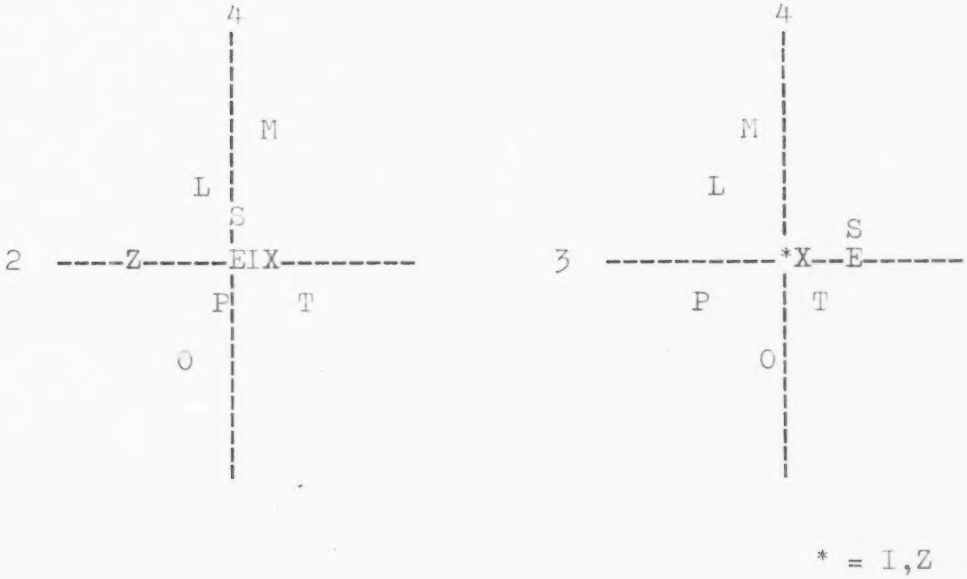


Figure 15

Plot of Multidimensional Scaling Solution
Eighth Grade Experimental Subjects Posttest
Mean Relatedness Coefficient Matrix

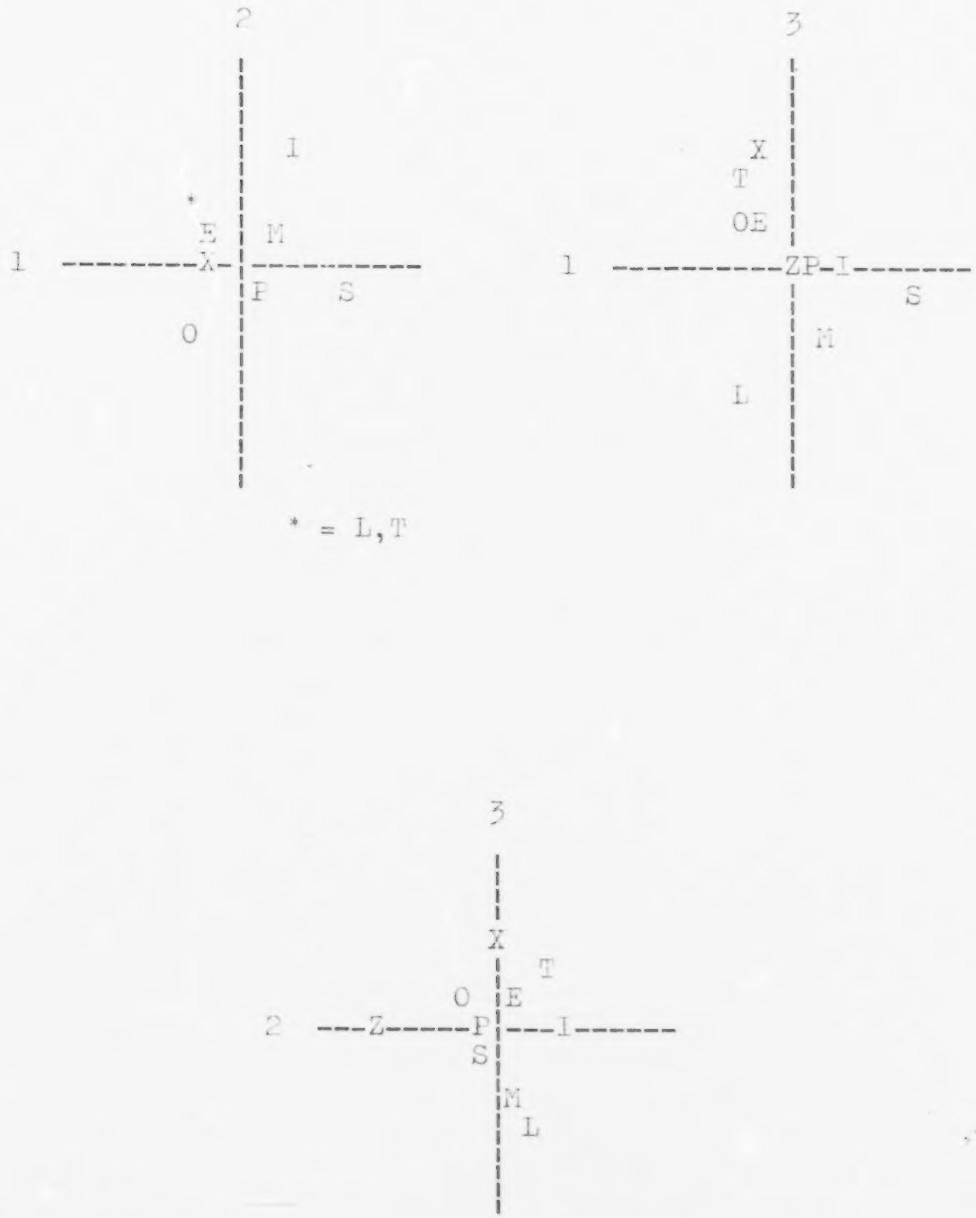
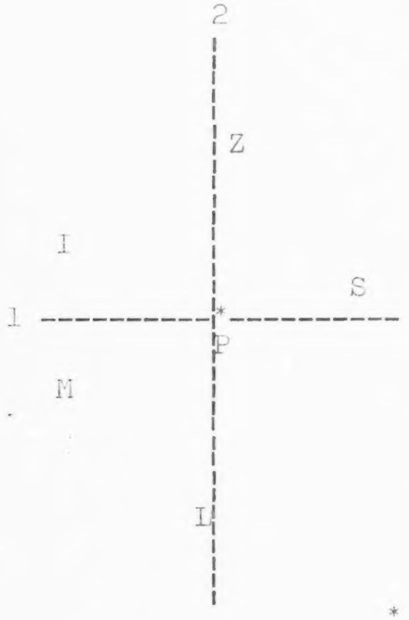


Figure 16

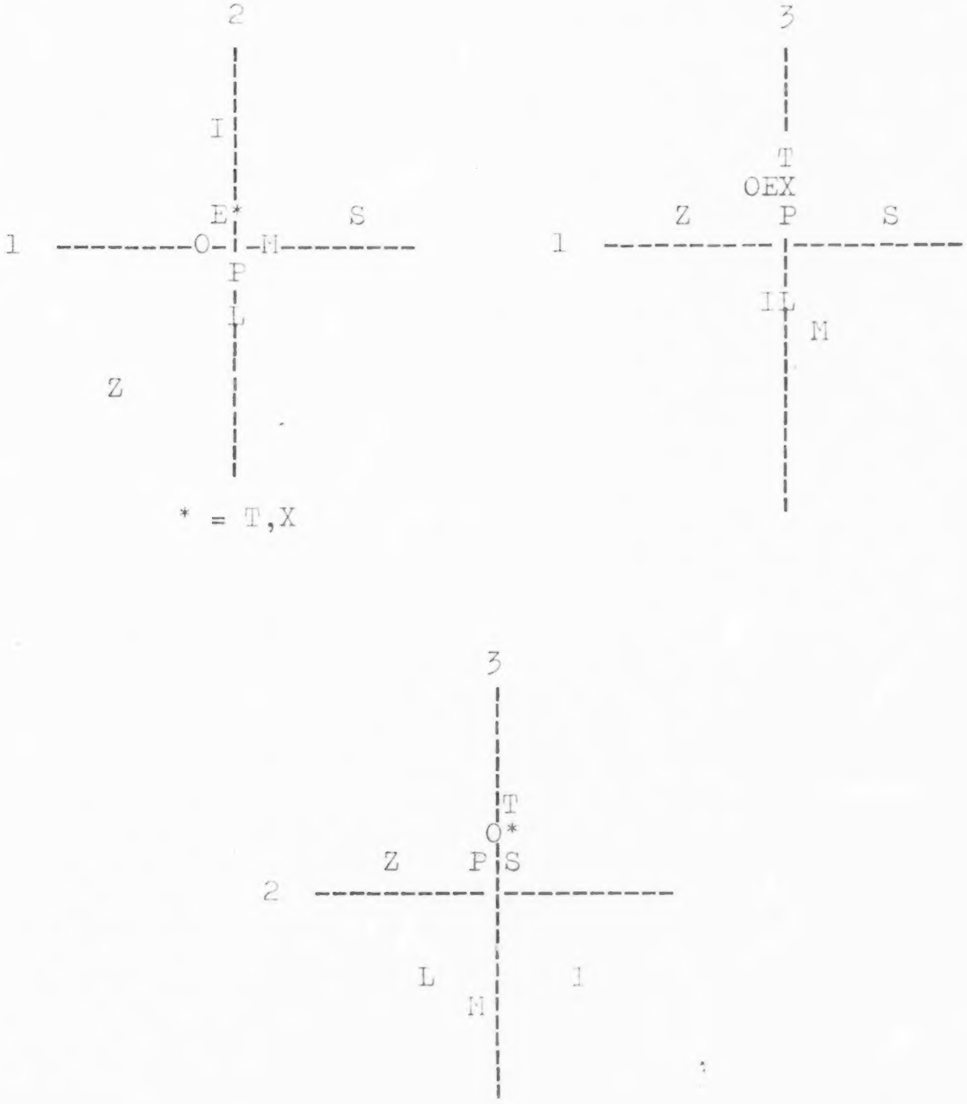
Plot of Multidimensional Scaling Solution
Eighth Grade Experimental Subjects Posttest
Median Relatedness Coefficient Matrix



* = E,T,X,O

Figure 17

Plot of Multidimensional Scaling Solution
Eighth Grade Experimental Subjects Retention
Mean Relatedness Coefficient Matrix



* = T,X

* = E,X

Figure 18

Plot of Multidimensional Scaling Solution
Eighth Grade Experimental Subjects Retention
Median Relatedness Coefficient Matrix

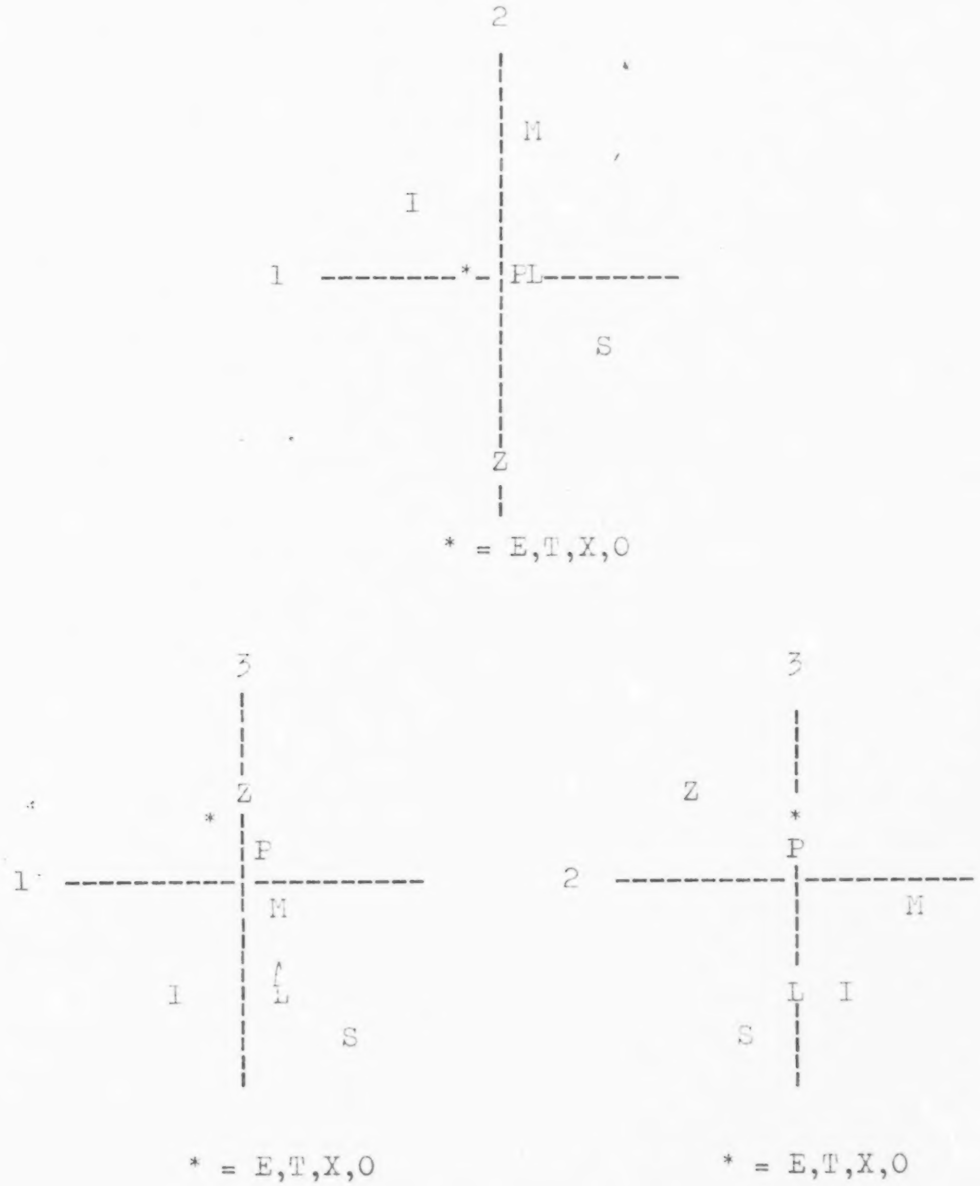
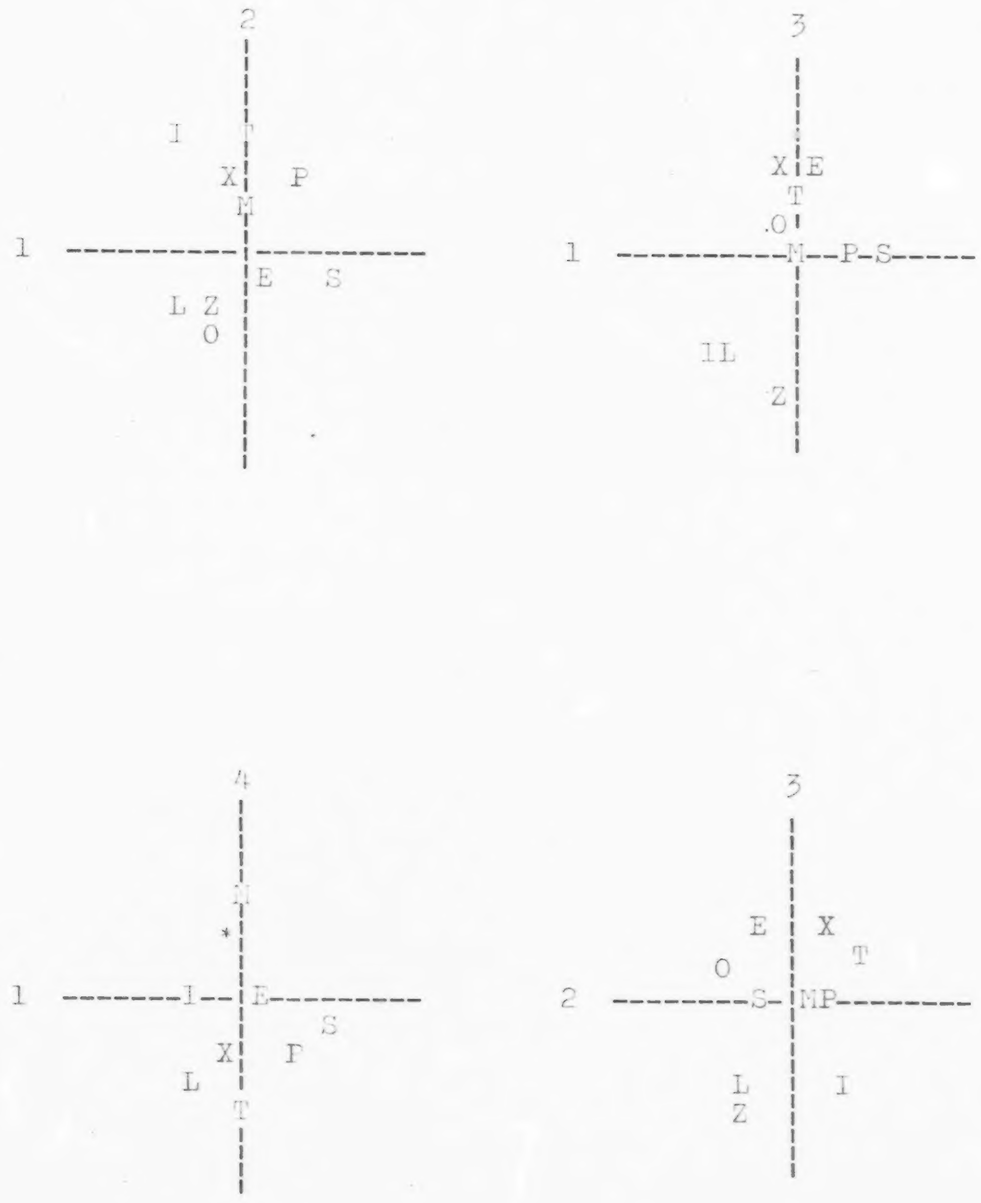


Figure 19

Plot of Multidimensional Scaling Solution
Eighth Grade Control Subjects Pretest
Mean Relatedness Coefficient Matrix



* = Z, O

Figure 19 (cont.)

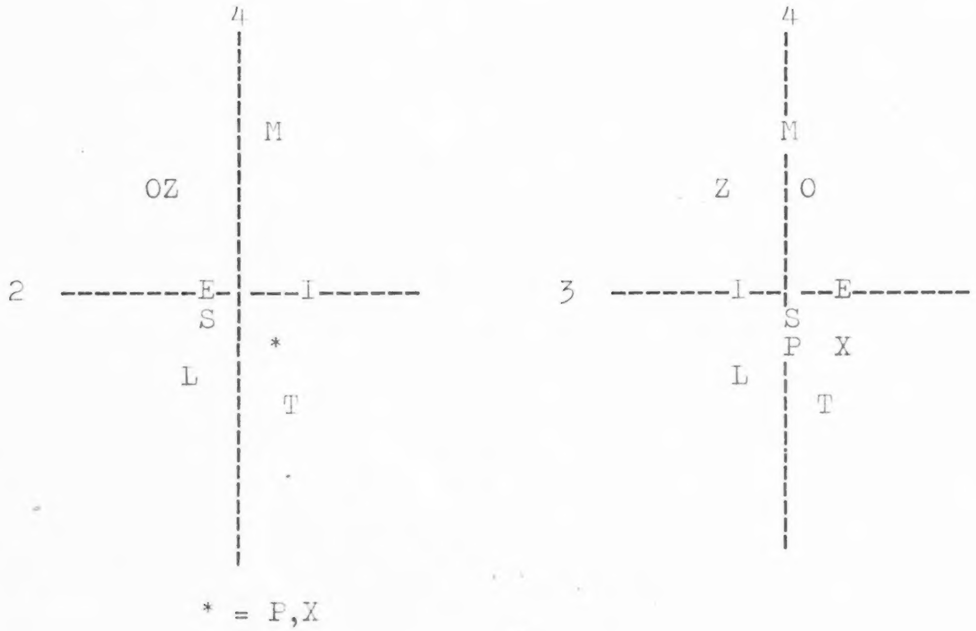


Figure 20

Plot of Multidimensional Scaling Solution
Eighth Grade Control Subjects Posttest
Mean Relatedness Coefficient Matrix

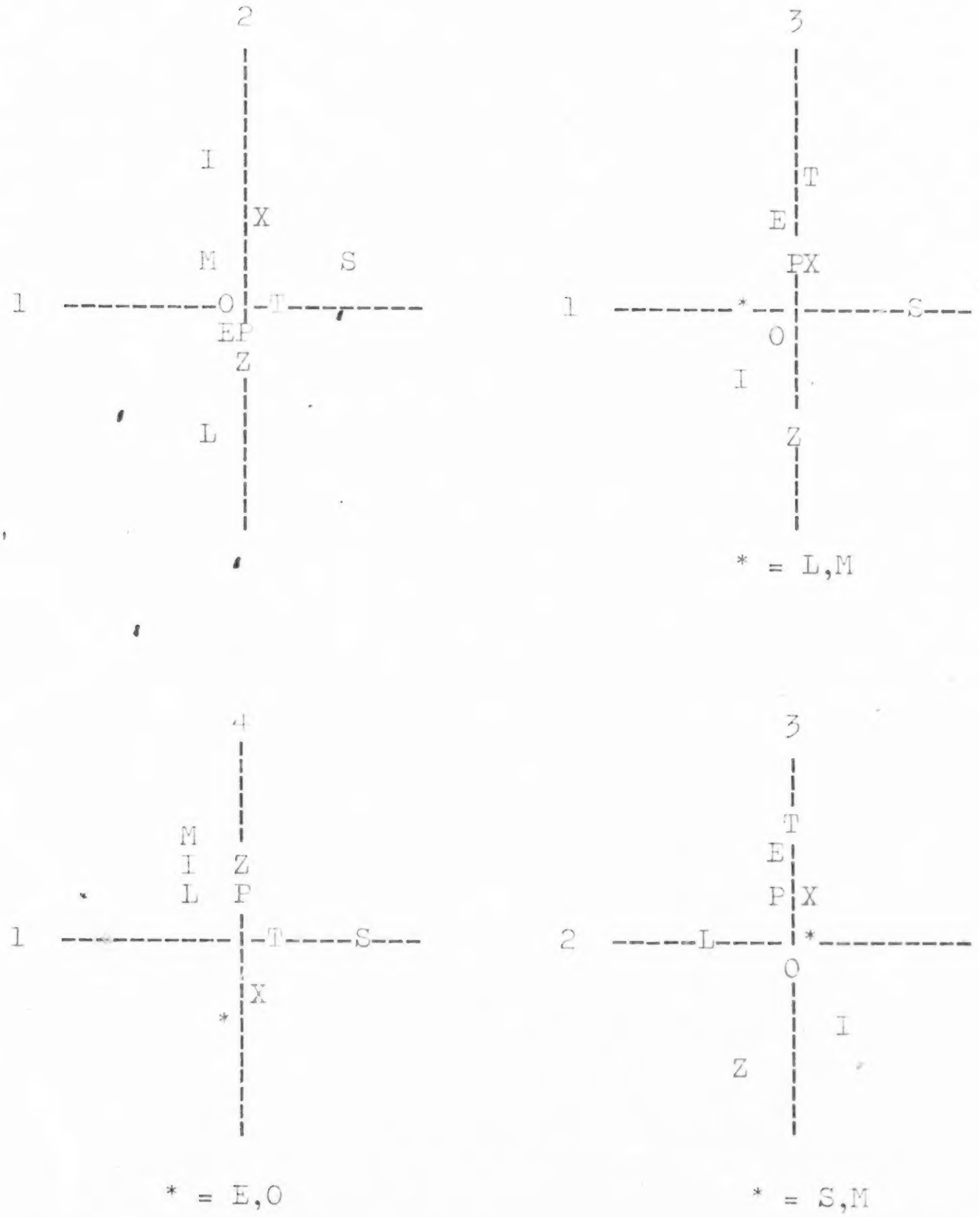


Figure 20 (cont.)

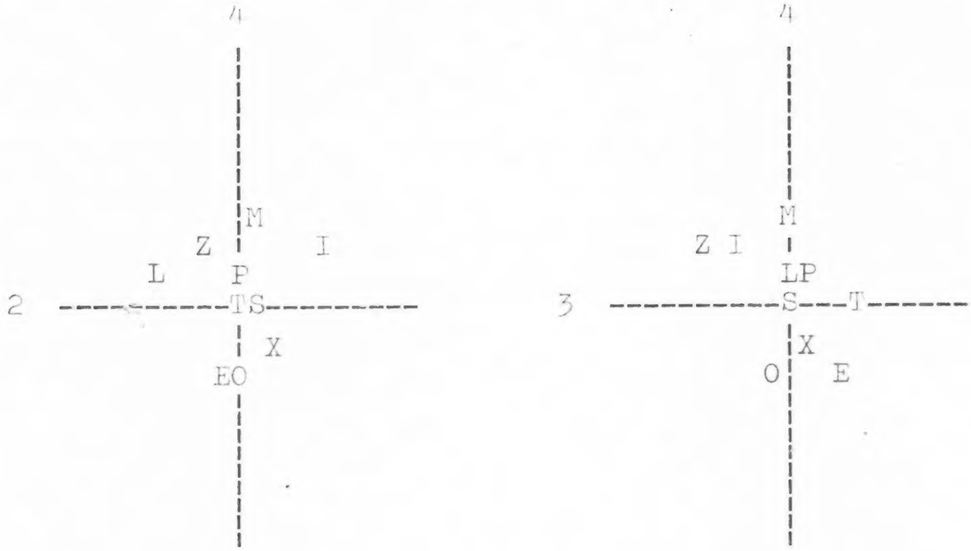


Figure 21

Plot of Multidimensional Scaling Solution
Eighth Grade Control Subjects Retention
Mean Relatedness Coefficient Matrix

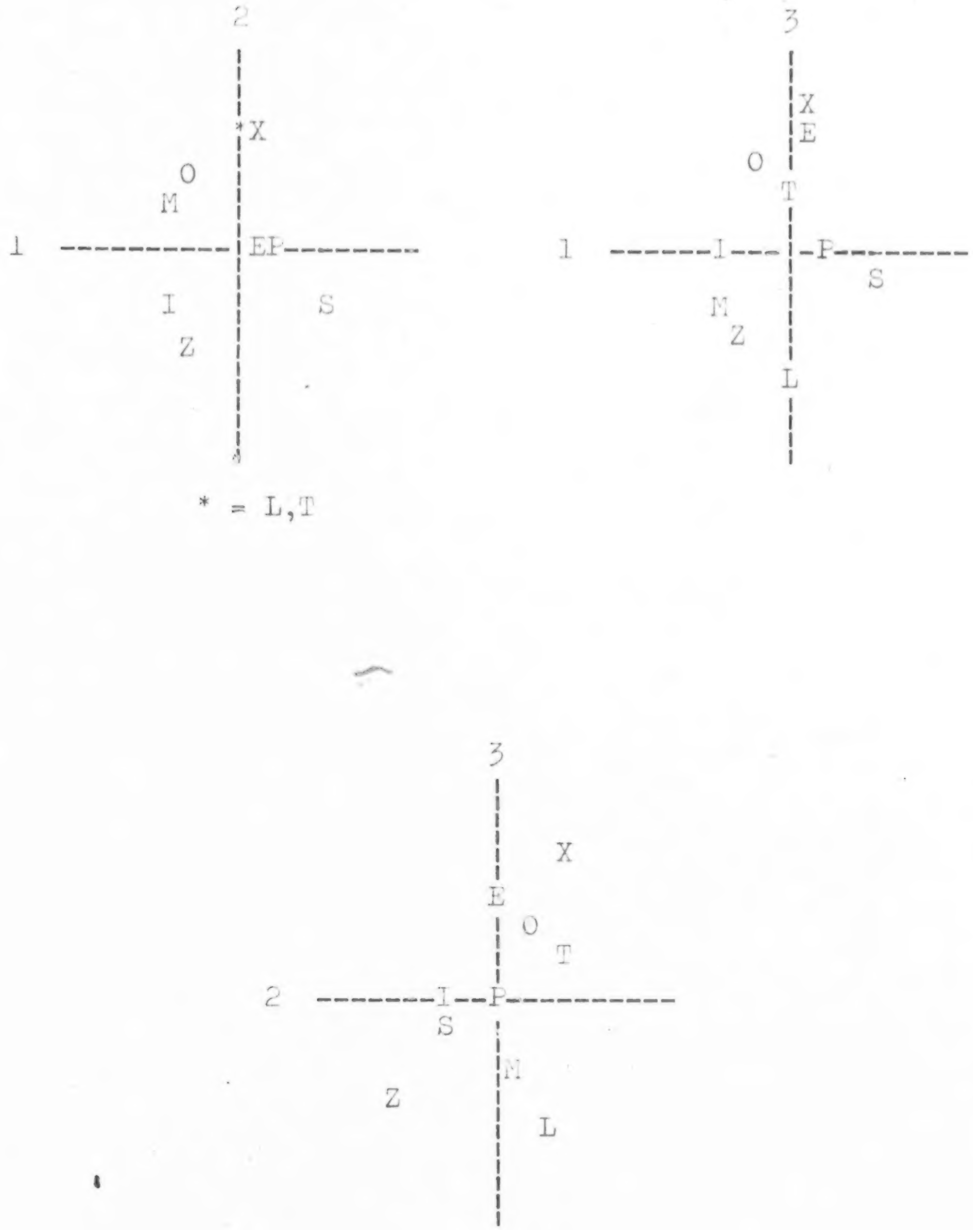
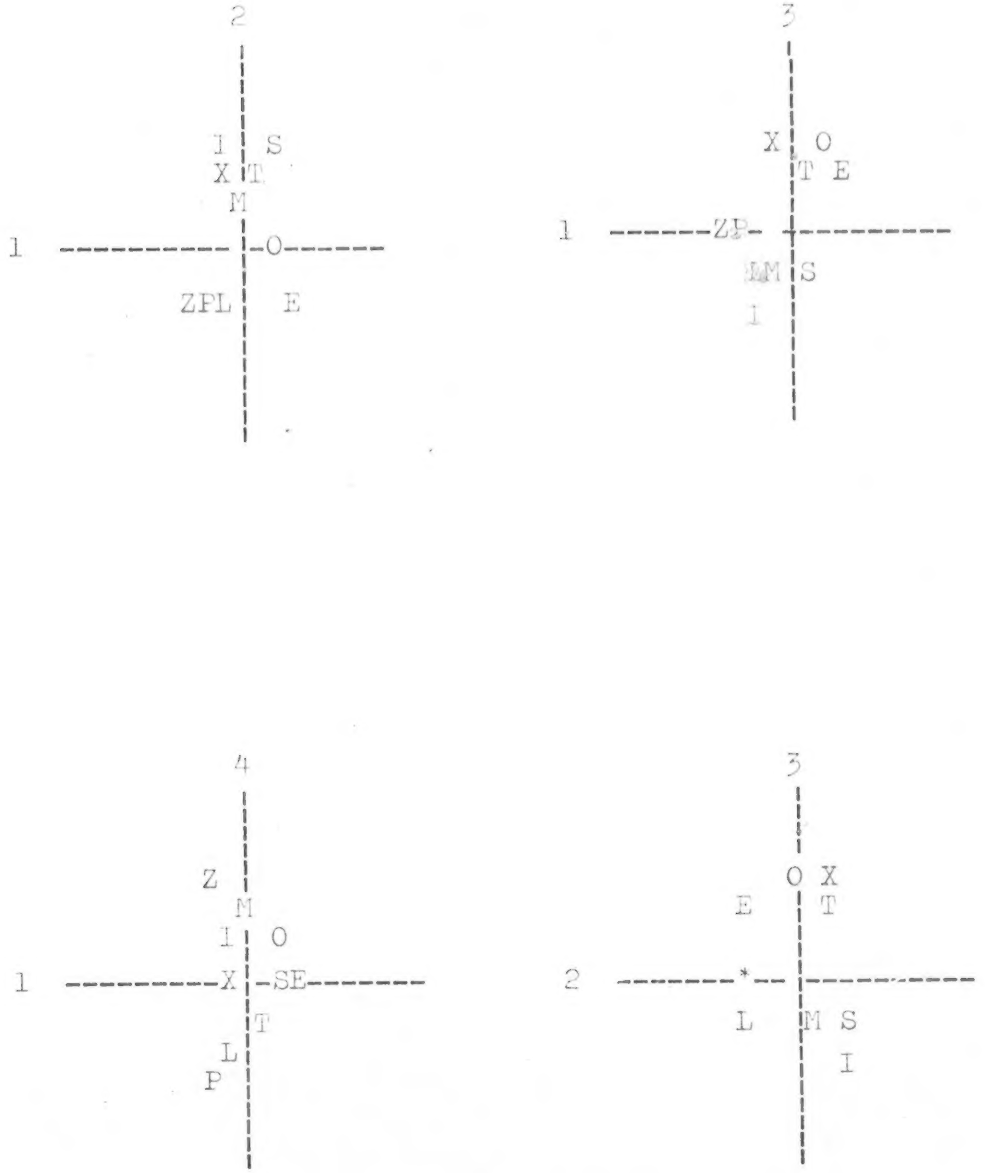


Figure 22

Plot of Multidimensional Scaling Solution
High School Experimental Subjects Pretest
Mean Relatedness Coefficient Matrix



* = P,Z

Figure 22 (cont.)

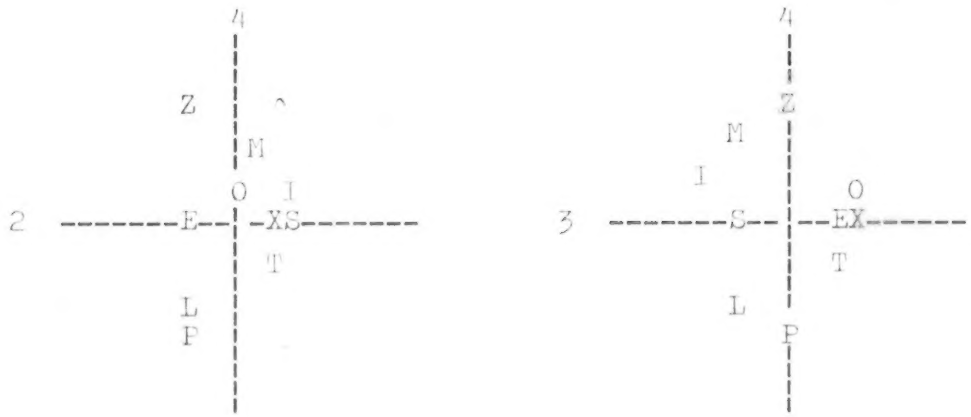


Figure 23

Plot of Multidimensional Scaling Solution
High School Experimental Subjects Posttest
Mean Relatedness Coefficient Matrix

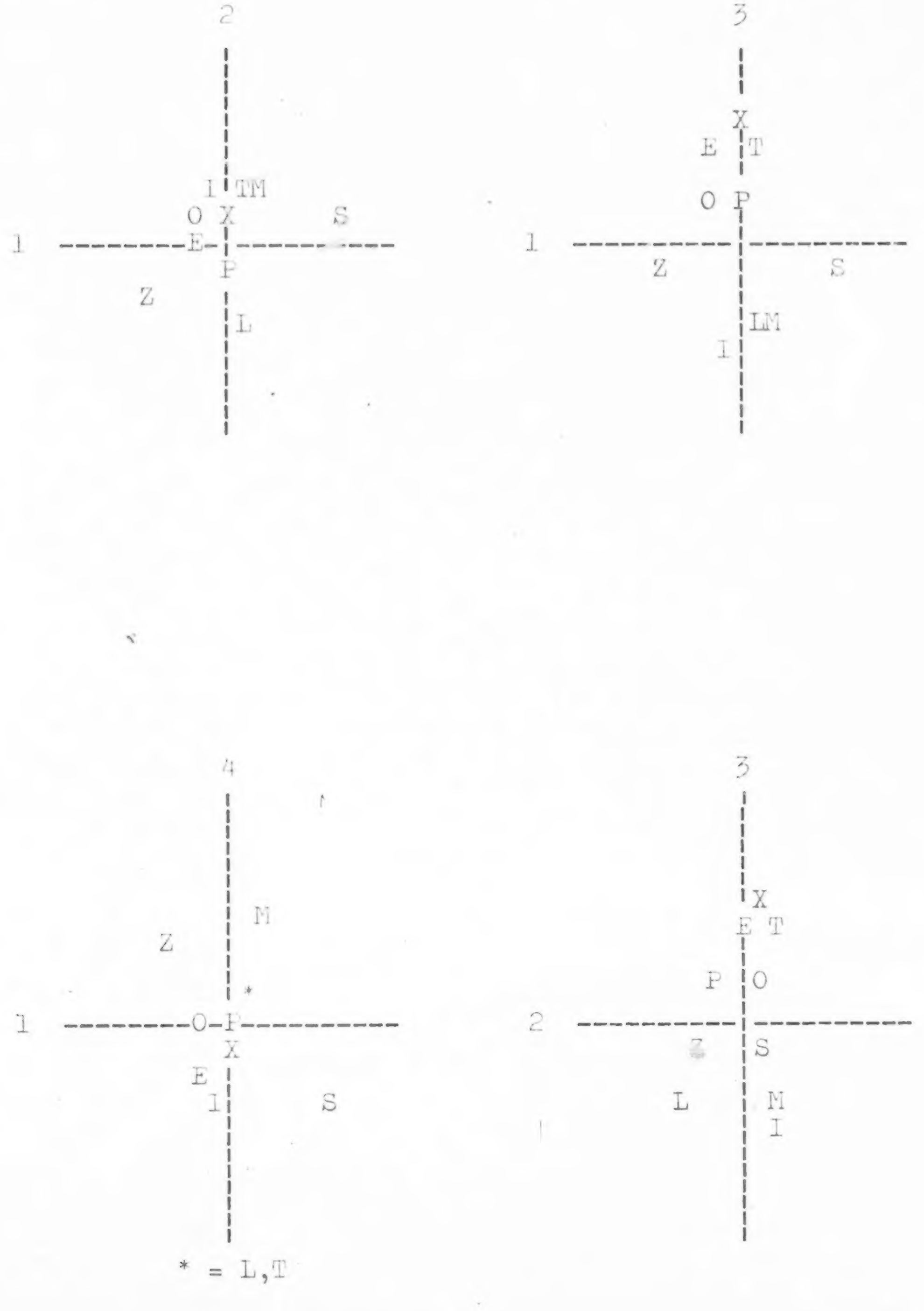
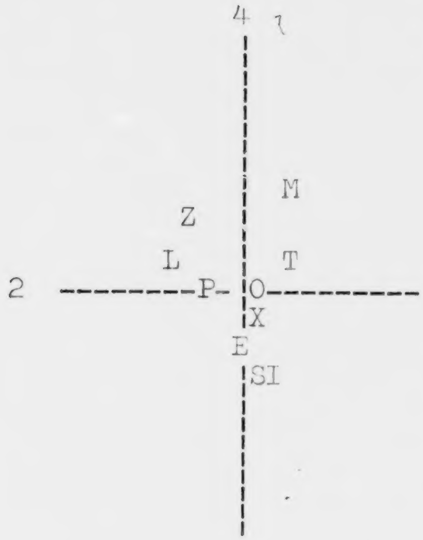
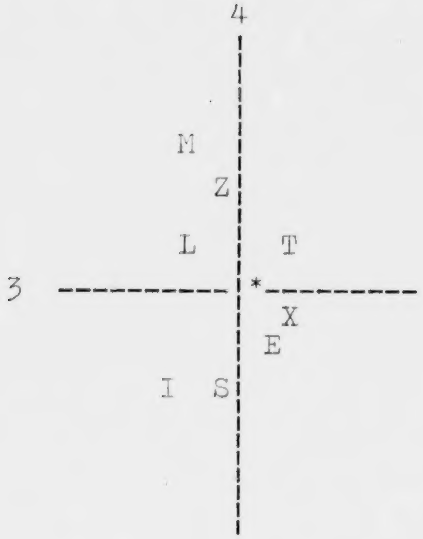


Figure 23 (cont.)





* = P,O

Figure 24

Plot of Multidimensional Scaling Solution
High School Experimental Subjects Posttest
Median Relatedness Coefficient Matrix

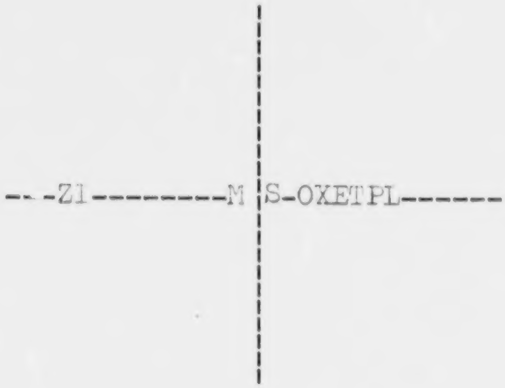


Figure 25

Plot of Multidimensional Scaling Solution
High School Experimental Subjects Retention
Mean Relatedness Coefficient Matrix

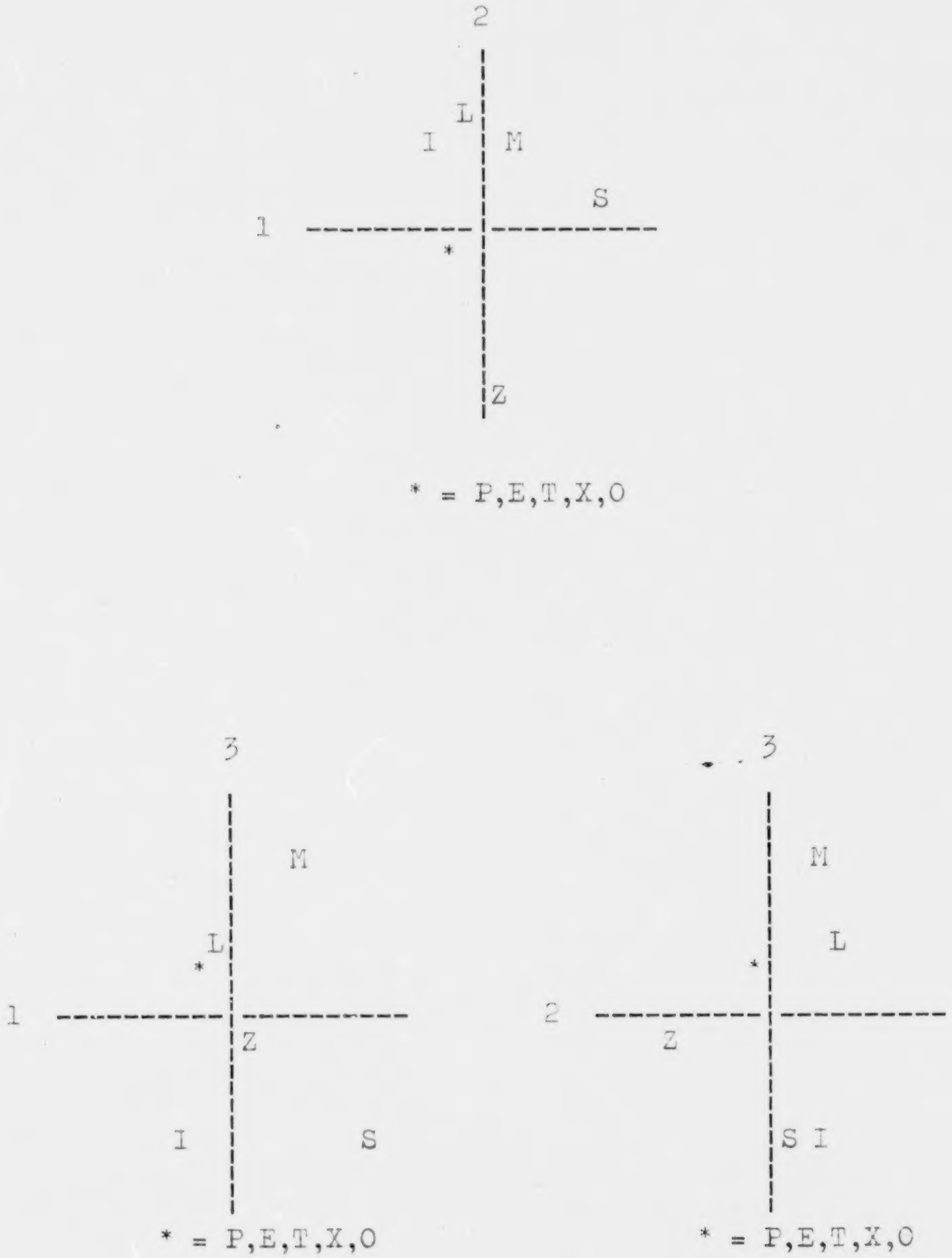
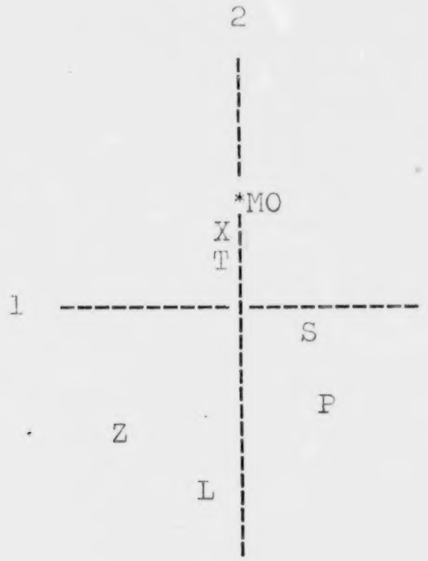


Figure 26

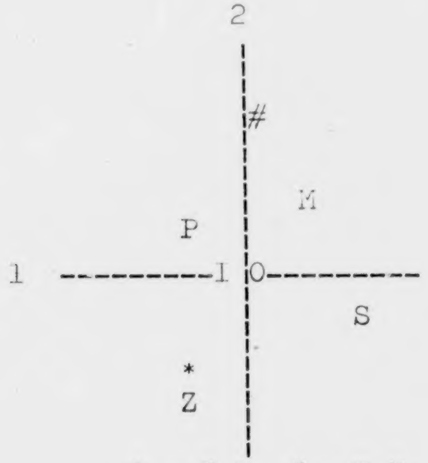
Plot of Multidimensional Scaling Solution
High School Experimental Subjects Retention
Median Relatedness Coefficient Matrix



* = I,E

Figure 27

Plot of Multidimensional Scaling Solution
High School Control Subjects Pre-
test Mean Relatedness Coefficient Matrix



* = E,L; # = T,X

Figure 28

Plot of Multidimensional Scaling Solution
High School Control Subjects Posttest
Mean Relatedness Coefficient Matrix

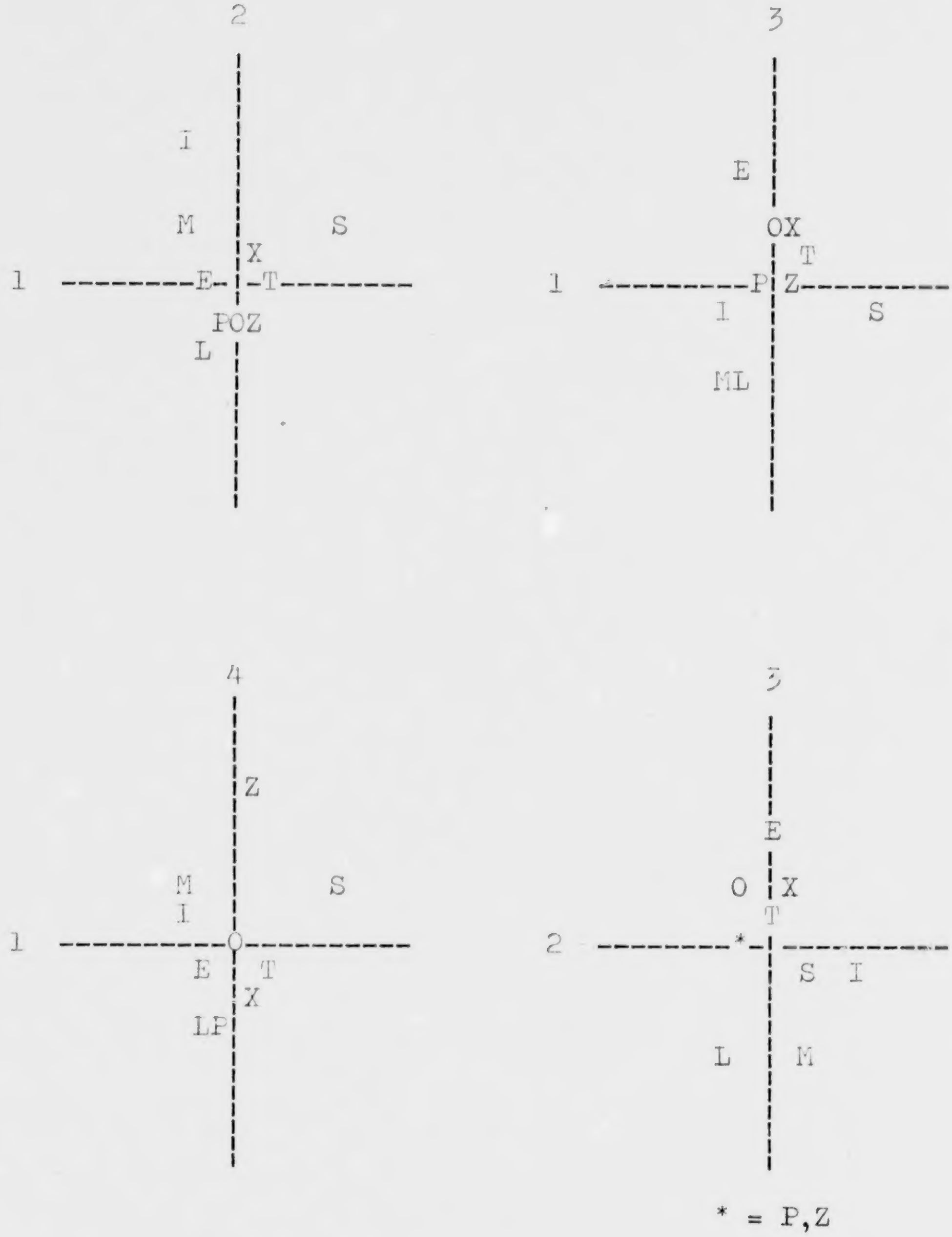


Figure 28 (cont.)

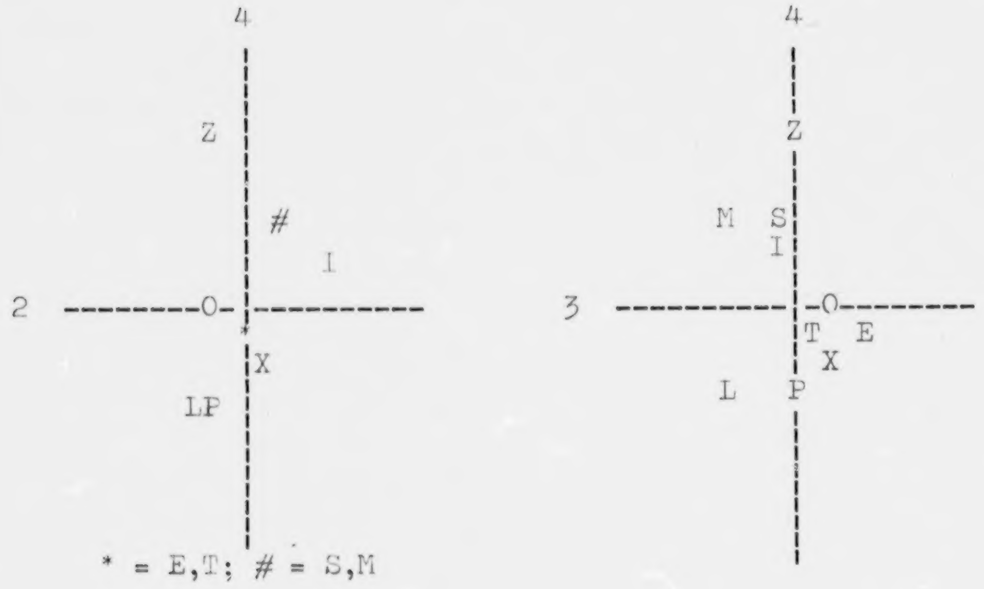


Figure 29

Plot of Multidimensional Scaling Solution
High School Control Subjects Retention
Mean Relatedness Coefficient Matrix

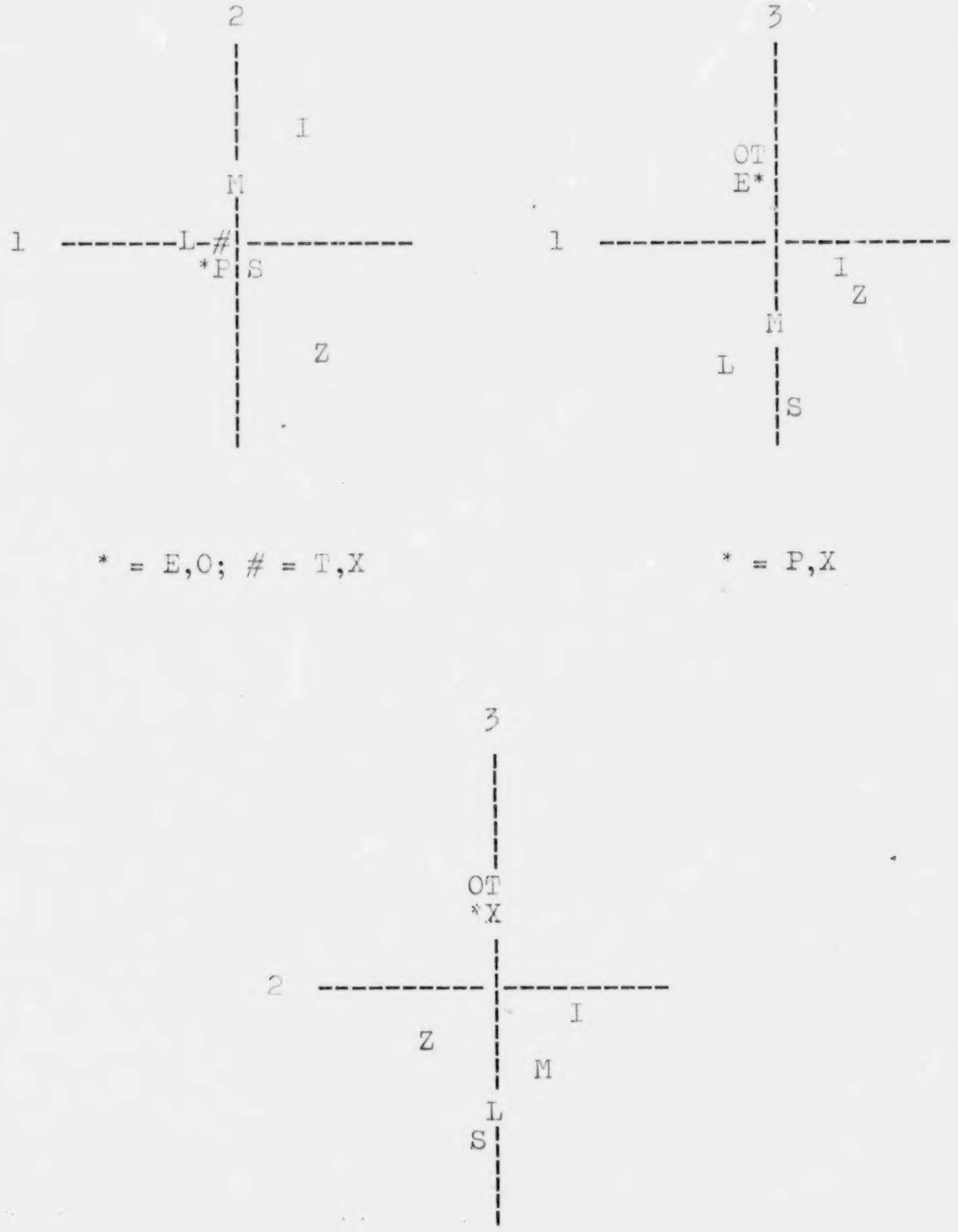
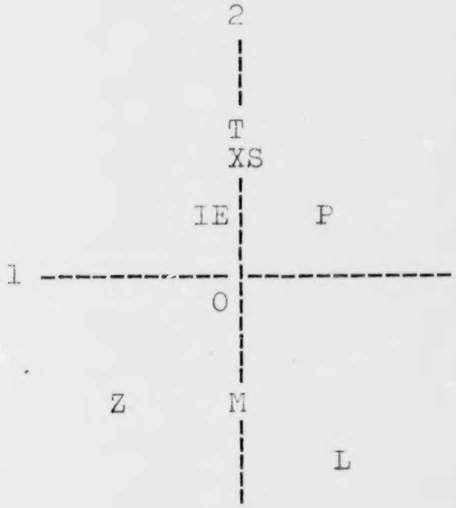


Figure 30

Plot of Multidimensional Scaling Solution
High School Control Subjects Retention
Median Relatedness Coefficient Matrix



Analysis of PC responses. The paragraph construction (PC) technique was used as an alternate measure to examine Ss' cognitive structures. Each paragraph was examined with a digraph analysis analagous to the analysis of content structure. (Trivial responses such as "event is outcome's cousin" were deleted.) Each S was asked to write one paragraph concerning each of five pairs of concepts. Five digraphs, corresponding to the five paragraphs, were combined to form a super-digraph (one for each S). Then, for each cell of the 2 x 3 x 1 (treatment by school level by test occasion) design, an element by element median was calculated and these median elements were combined to form a PC distance matrix. (It should be noted that each median entry was obtained from a different N, depending on the number of Ss who gave a response corresponding to that particular entry.) Although each S was required to discuss the relationships between only five pairs of concepts, at least some Ss in each experimental group found it necessary to include other relationships and thus no infinite elements were found in the PC distance matrices. Finally, the PC matrices were converted to a similarity matrix in the same manner as for the digraph (see Tables 13-17). Scaling solutions for each PC similarity matrix are presented in Figures 31-35 and the numerical results of Kruskal's (1964) procedure are found in Appendix B. (The sixth grade control group PC matrix is not given due to the fact that no usuable responses were obtained.)

Table 13

Paragraph Construction Similarity Matrix for Sixth Grade Experimental Ss

	Prb.	Ind.	Event	Zero	E.L.	Int.	Tr.	Exp.	M.E.	Out.
Prb.	1.0	2500	5000	1818	2500	2500	4000	3333	2222	3333
Ind.	2000	1.0	2222	1667	2500	2500	2500	3333	2500	3333
Event	5000	2000	1.0	3333	2500	3333	2500	3333	3333	4000
Zero	2000	3333	2500	1.0	3333	2500	2000	3333	2222	3333
E.L.	2500	2500	3333	3333	1.0	3333	2222	2500	3333	3333
Int.	2857	2000	3333	1429	3333	1.0	3333	2500	2000	3333
Tr.	3333	2500	3333	1250	2222	3333	1.0	5000	2000	3333
Exp.	3333	3333	3333	2222	2500	2500	5000	1.0	2857	5000
M.E.	2000	3333	2222	1667	2857	2000	2000	3333	1.0	3333
Out.	3333	4000	4000	3333	3333	3333	3333	5000	3333	1.0

*decimals omitted

Table 14

Paragraph Construction Similarity Matrix for Eighth Grade Experimental Ss

	Prb.	Ind.	Event	Zero	E.L.	Int.	Tr.	Exp.	M.E.	Out.
Prb.	1.0	2500	2857	2857	2500	2000	2500	3333	2500	3333
Ind.	2500	1.0	2500	2857	3333	2500	3333	2500	3333	3333
Event	3333	2500	1.0	3333	2500	3333	3333	3333	2500	5000
Zero	3333	2500	3333	1.0	5000	2000	2500	2500	2000	5000
E.L.	2222	2500	2857	5000	1.0	1818	2500	2222	3333	3333
Int.	2222	2500	3333	2222	2000	1.0	2857	2500	2000	2000
Tr.	2500	3333	2857	2500	1667	3333	1.0	5000	1818	2500
Exp.	3333	2500	3333	2857	2222	2857	5000	1.0	2000	3333
M.E.	2500	2500	2500	2500	3333	2000	2222	2500	1.0	3333
Out.	2500	3333	3333	4000	3333	2222	3333	3333	3333	1.0

* decimals omitted

Table 15

Paragraph Construction Similarity Matrix for Eighth Grade Control Ss

	Prb.	Ind.	Event	Zero	E.L	Int.	Tr.	Exp.	M.E.	Out.
Prb.	1.0	4000	3333	3333	0	1429	2500	3333	0	2500
Ind.	3333	1.0	0	0	0	0	3333	3333	0	2857
Event	3333	3333	1.0	3333	0	3333	5000	3333	0	3333
Zero	0	0	0	1.0	5000	0	3333	3333	0	5000
E.L.	0	0	0	5000	1.0	0	0	0	3333	0
Int.	1429	0	3333	0	0	1.0	3333	2000	0	3333
Tr.	2500	5000	5000	3333	0	3333	1.0	5000	2000	3333
Exp.	4000	3333	3333	3333	0	2000	5000	1.0	2500	5000
M.E.	2500	0	0	0	0	0	2000	2500	1.0	3333
Out.	3333	3333	4000	5000	0	3333	3333	5000	3333	1.0

* decimals omitted

Table 16

Paragraph Construction Similarity Matrix for High School Experimental Ss

	Prb.	Ind.	Event	Zero	E.L.	Int.	Tr.	Exp.	M.E.	Out.
Prb.	1.0	3333	5000	4000	3333	2500	2500	5000	2857	3333
Ind.	2500	1.0	2500	2857	1429	2000	5000	3333	3333	5000
Event	5000	2500	1.0	2500	3333	3333	3333	3333	3333	3333
Zero	5000	3333	2500	1.0	2857	2222	2500	3333	2000	5000
E.L.	3333	2000	3333	2500	1.0	2000	2000	2500	2857	3333
Int.	2500	2000	3333	2000	2000	1.0	2000	2857	2500	2500
Tr.	3333	5000	2857	2500	2000	2000	1.0	5000	2000	3333
Exp.	5000	3333	3333	3333	2500	2857	5000	1.0	3333	5000
M.E.	2857	3333	3333	2500	2857	2500	2000	2500	1.0	5000
Out.	3333	5000	3333	4000	3333	3333	3333	5000	4000	1.0

* decimals omitted

Table 17

Paragraph Construction Similarity Matrix for High School Control Ss

	Prb.	Ind.	Event	Zero	E.L.	Int.	Tr.	Exp.	M.E.	Out.
Prb.	1.0	1818	5000	2500	5000	2500	2500	5000	0	3333
Ind.	2222	1.0	1667	2500	1667	2500	3333	2500	0	2857
Event	3333	2222	1.0	3333	3333	3333	3333	3333	2000	3333
Zero	2857	2857	3333	1.0	3333	3333	2857	5000	0	5000
E.L.	2000	1111	3333	2000	1.0	3333	2000	1667	3333	1429
Int.	2000	1818	2222	2222	0	1.0	3333	3333	2000	2000
Tr.	3333	2500	2500	2500	2500	2857	1.0	5000	2000	3333
Exp.	5000	2500	2857	4000	1667	3333	5000	1.0	2000	5000
M.E.	1250	1111	0	1250	3333	3333	2000	1667	1.0	3333
Out.	2857	2857	2500	5000	2000	5000	3333	5000	3333	1.0

*decimals omitted

Figure 31

Plot of Multidimensional Scaling Solution
Paragraph Construction Data
Sixth Grade Experimental Subjects

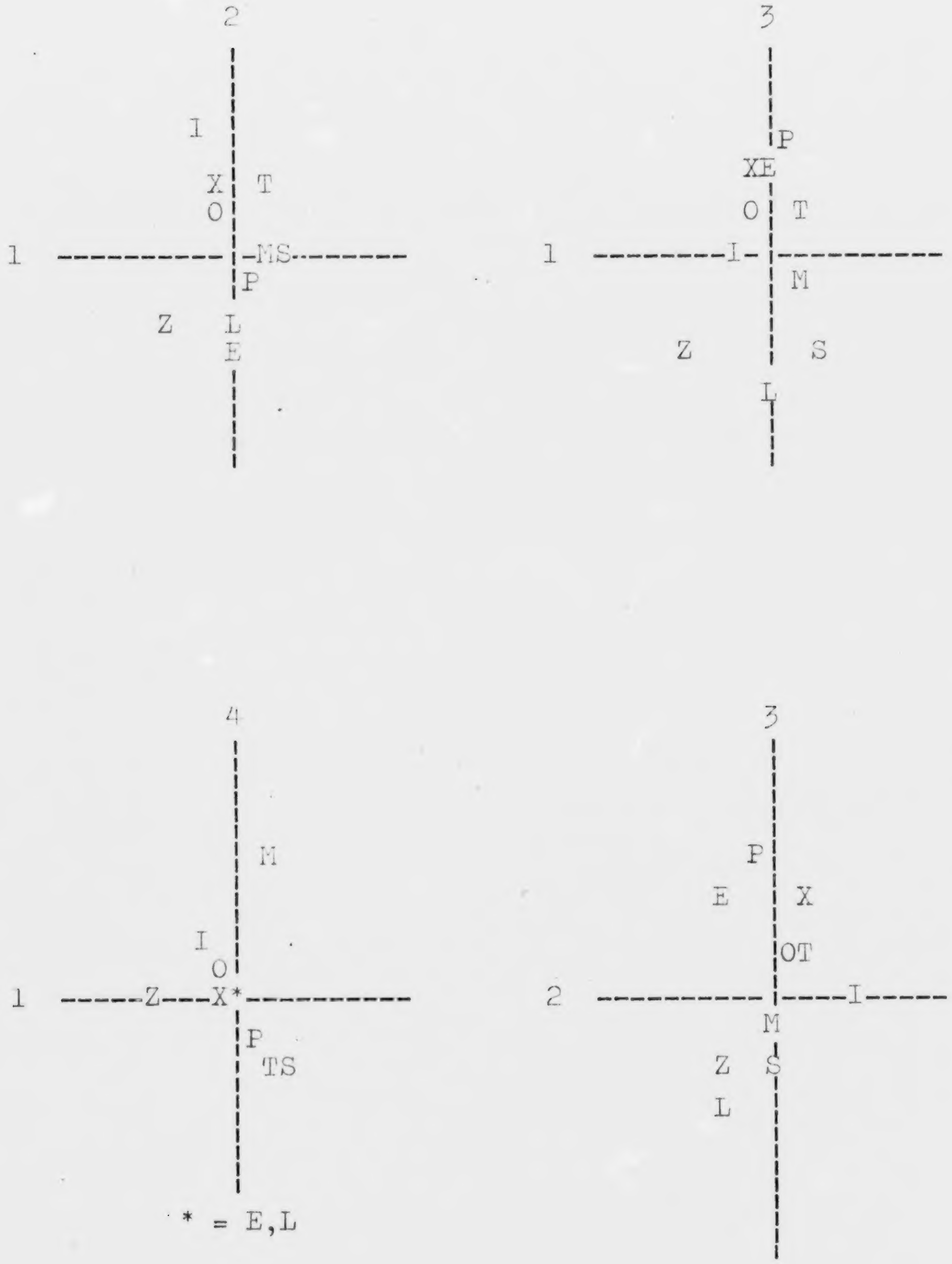
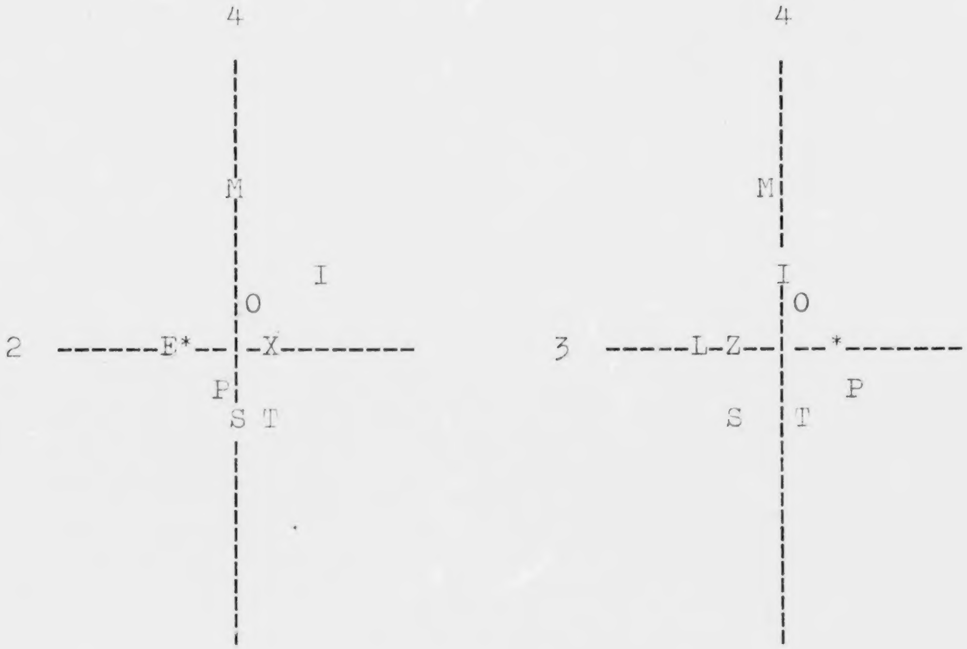


Figure 31 (cont.)



* = Z,L

* = E,X

Figure 32

Plot of Multidimensional Scaling Solution
Paragraph Construction Data
Eighth Grade Experimental Subjects

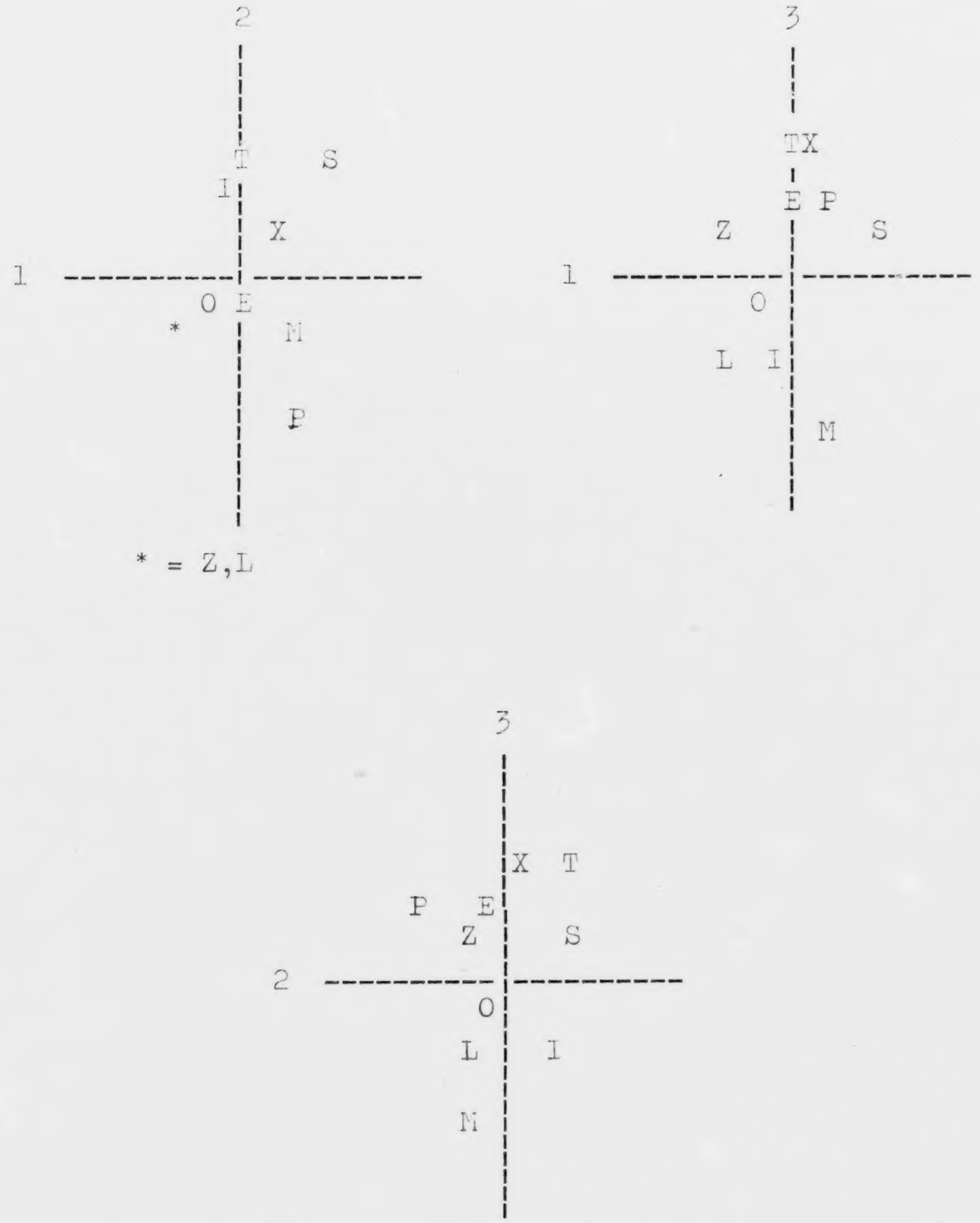


Figure 33

Plot of Multidimensional Scaling Solution
Paragraph Construction Data
Eighth Grade Control Subjects

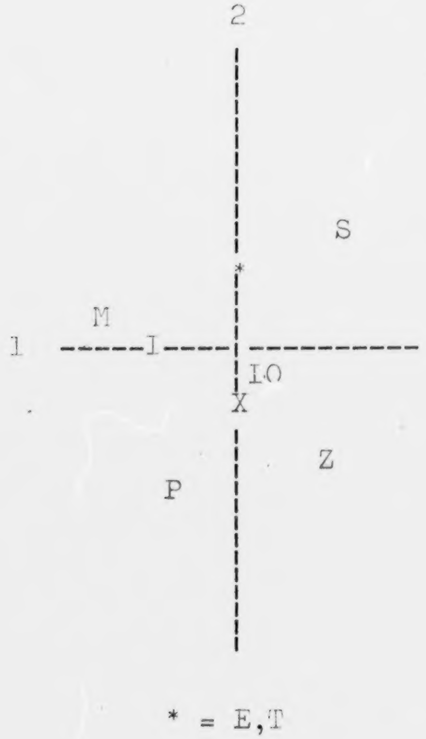


Figure 34

Plot of Multidimensional Scaling Solution
Paragraph Construction Data
High School Experimental Subjects

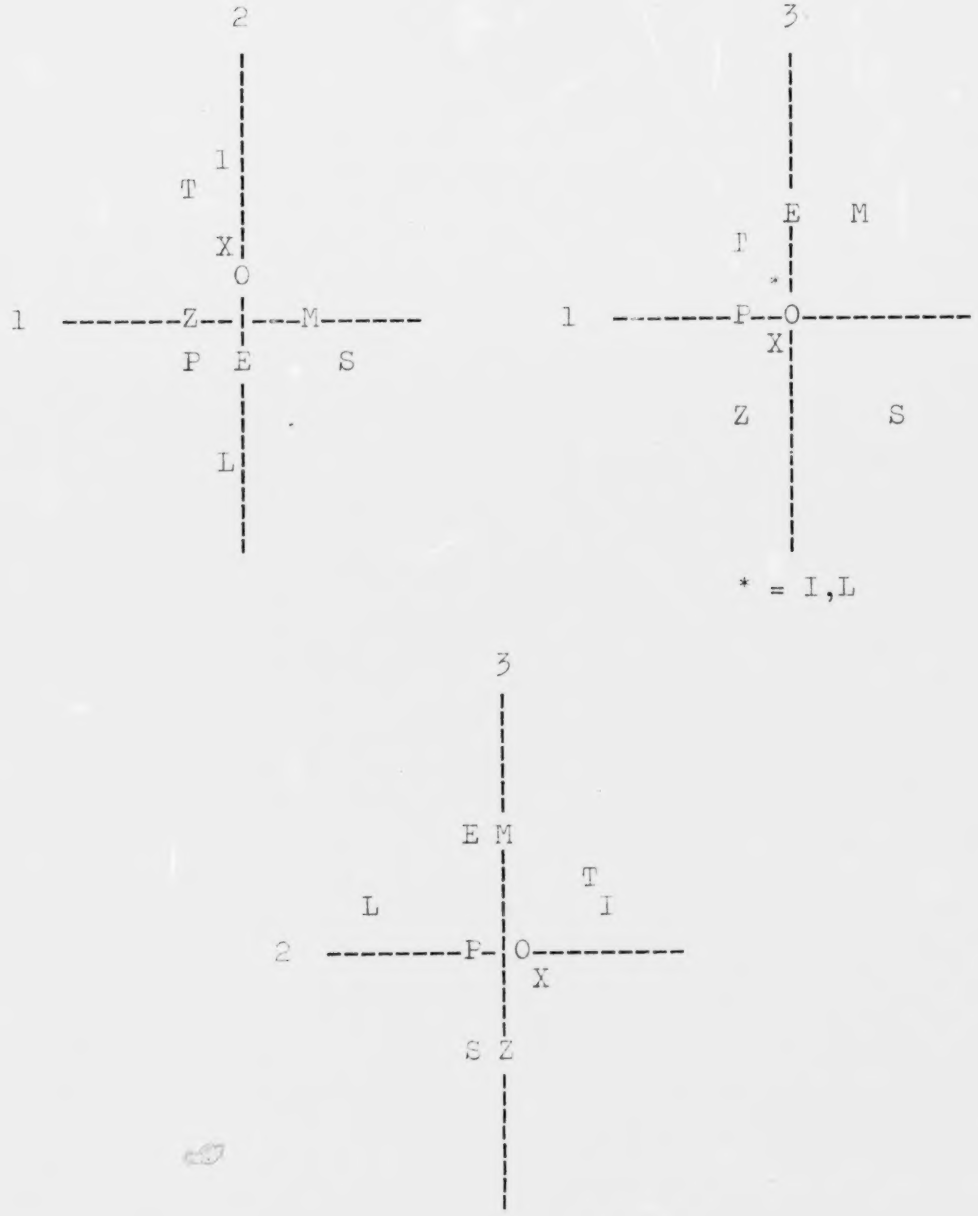


Figure 35

Plot of Multidimensional Scaling Solution
Paragraph Construction Data
High School Control Subjects

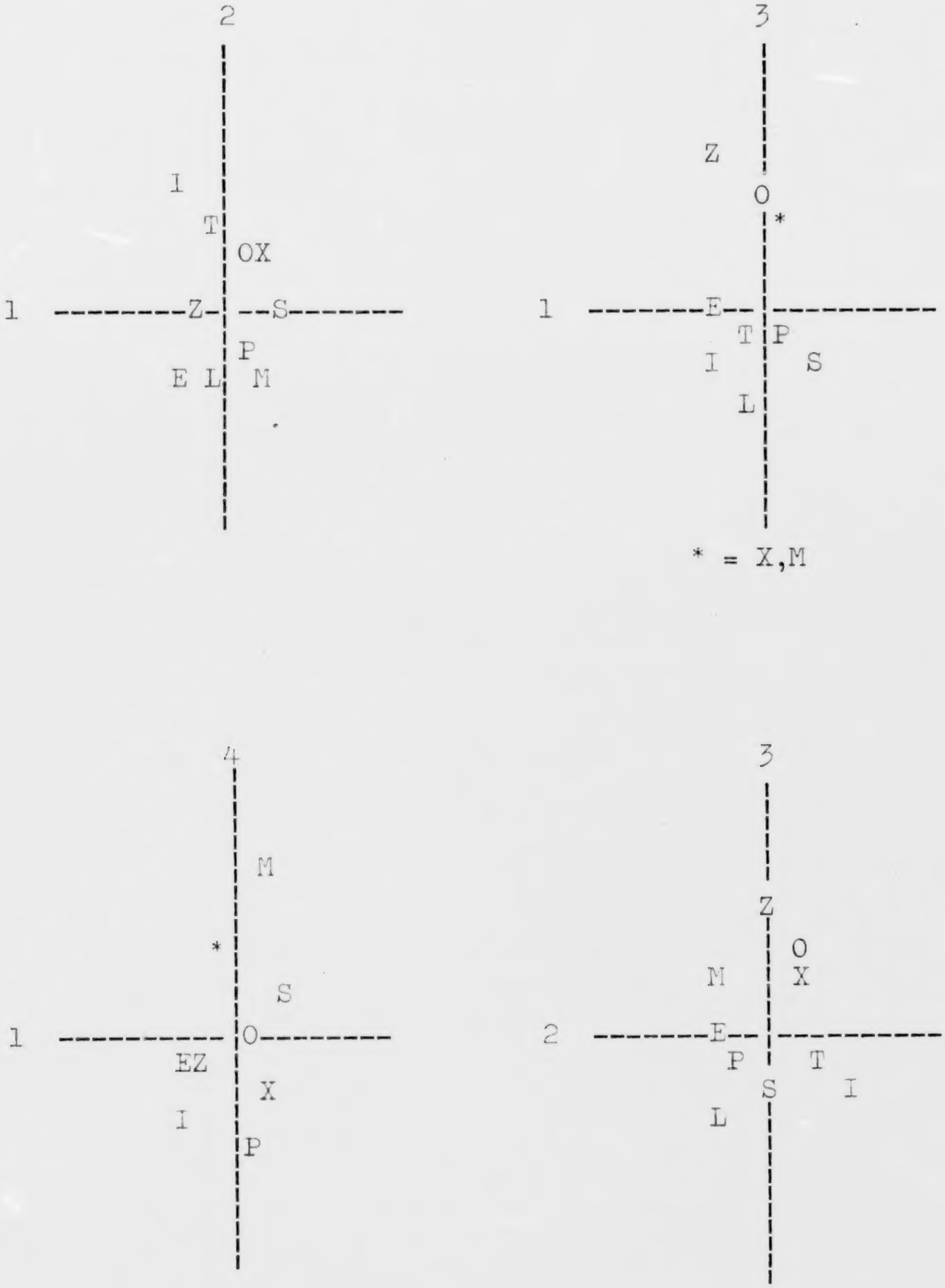
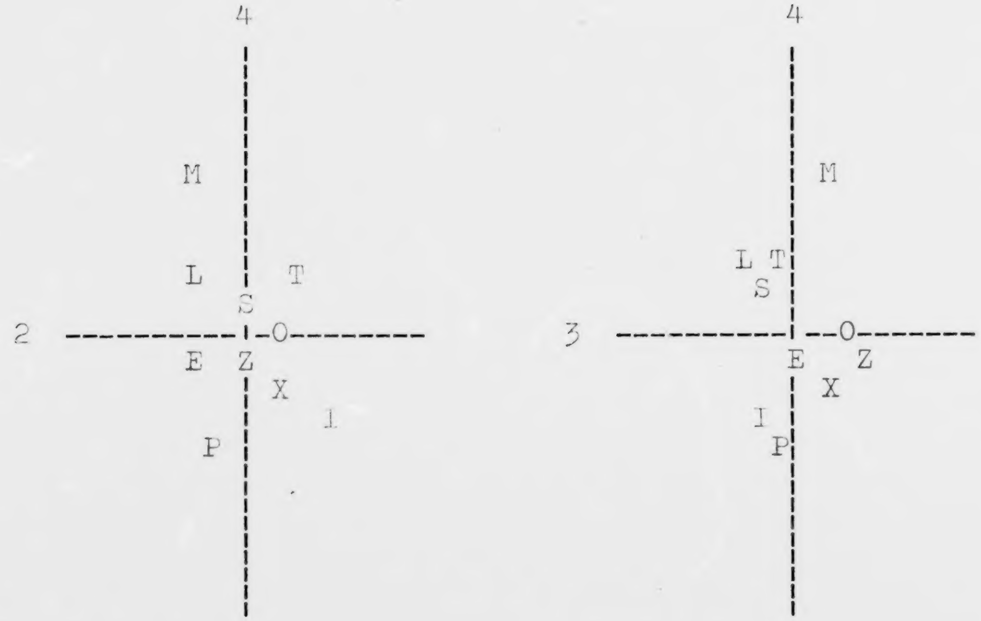


Figure 35 (cont.)



Comparison of WA, Achievement, Ability, and Attitude Data

A correlational analysis of the data obtained from each experimental group was performed to investigate the relationships among various types of data gathered on Ss. Due to the quite different types of data gathered, two intercorrelation matrices were obtained. Tables 18-23 present product-moment intercorrelation matrices and non-parametric (Kendall Tau rank correlation) intercorrelation matrices. The reason for obtaining two types of correlations is that one can reasonably expect IQ and achievement, for example, to come from a bivariate normal distribution and thus the product-moment correlation is appropriate. On the other hand, there is no prior evidence to indicate that the Euclidean scores (from the WA data) are normally distributed and one might wish to use the more conservative Kendall correlation. PC data was not analyzed by subjects with an Euclidean distance score and thus is not included in the correlation analyses. Euclidean distance scores, explained in the next section, are a measure of how well Ss' cognitive structures corresponded to the content structure. Perfect correspondence between achievement data and WA data would be indicated by a correlation of -1.0 since a smaller Euclidean distance score implies a closer relationship between content structure and cognitive structure.

Table 18**

Product Moment Correlations
Sixth Grade Experimental Group

	Att. Math.	Pre Ach.	Post Ach.	Ret Ach.	Pre Graph	Pre Digr.	Post Graph
Att. Math	1.000 (32)	126 (31)	195 (30)	084 (23)	-023 (32)	-024 (32)	-078 (30)
Pre Ach		1.000 (31)	434* (30)	249 (22)	156 (31)	160 (31)	-334 (30)
Post Ach			1.000 (31)	721* (22)	-081 (30)	-077 (30)	-442* (31)
Ret Ach				1.000 (23)	050 (23)	051 (23)	-041 (22)
Pre Gra					1.000 (32)	1.000* (32)	494* (30)
Pre Dig						1.000 (32)	488* (30)
Post Gra							1.000 (31)

	Post Digr.	Ret. Graph	Ret. Digr.	L/T Verb.	L/T N.V.	L/T Total	CTBS Read.	CTBS Arith.
Att. Math	-071 (30)	-546* (22)	-547* (22)	-209 (30)	-195 (30)	-213 (30)	-139 (30)	-021 (30)
Pre Ach	-343 (30)	-257 (21)	-263 (21)	275 (29)	311 (29)	308 (29)	206 (29)	353 (29)
Post Ach	-460* (31)	-457* (21)	-472* (21)	684* (28)	596* (28)	674* (28)	665* (28)	747* (28)
Ret Ach	-061 (22)	-238 (22)	-246 (22)	673* (22)	659* (22)	722* (22)	627* (22)	722* (22)
Pre Gra	463* (30)	369 (22)	325 (22)	-077 (30)	-206 (30)	-150 (30)	-020 (30)	-099 (30)
Pre Dig	457* (30)	364 (22)	320 (22)	-072 (30)	-205 (30)	-148 (30)	023 (30)	-093 (30)
Post Gra	998* (31)	656* (21)	633* (21)	-255 (28)	-249 (28)	-261 (28)	-250 (28)	-239 (28)

Table 18 (cont.)

	Post Digr.	Ret. Graph	Ret. Digr.	L/T Verb.	L/T N.V.	L/T Total	CTBS Read.	CTBS Arith.
Post Digr	1.000 (31)	658* (21)	638* (21)	-261 (28)	-245 (28)	-263 (28)	-263 (28)	-248 (28)
Ret Gra		1.000 (22)	998* (22)	004 (21)	-042 (21)	-017 (21)	-036 (21)	-087 (21)
Ret Digr			1.000 (22)	002 (21)	-030 (21)	-011 (21)	-047 (21)	-094 (21)
L/T Verb				1.000 (30)	801* (30)	945* (30)	899* (30)	772* (30)
L/T N.V.					1.000 (30)	953* (30)	728* (30)	705* (30)
L/T Tot						1.000 (30)	853* (30)	776* (30)
CTBS Read							1.000 (30)	776* (30)
CTBS Arith.								1.000 (30)

(N's in parenthesis)

*$p < .05$

**decimals omitted

Table 19**

Rank Order Correlations
Sixth Grade Experimental Group

	Att. Math.	Pre Ach.	Post Ach.	Ret. Ach.	Pre Graph	Pre Digr.	Post Graph
Att. Math	1.0	144	088	-025	103	114	-031
Pre Ach		1.0	272*	141	025	027	-136
Post Ach			1.0	471*	-222	-214	-306*
Ret Ach				1.0	-094	-089	-044
Pre Gra					1.0	992*	540*
Pre Dig						1.0	534*
Post Gra							1.0

	Post Digr.	Ret. Graph	Ret. Digr.	L/T Verb.	L/T N.V.	L/T Total	CTBS Read.	CTBS Arith.
Att Math	-024	-417*	-406*	-162	-115	-154	-094	-012
Pre Ach	-113	-125	-125	188	225	227	201	248
Post Ach	-314*	-133	-132	421*	379*	428*	439*	503*
Ret Ach	-058	-143	-142	598*	503*	537*	481*	583*
Pre Gra	558*	202	201	-215	-396*	-328*	-117	-202
Pre Dig	552*	184	183	-215	-395*	-327*	-116	-201
Post Gra	984*	543*	550*	-196	-231	-230	-230	-222

Table 19 (cont.)

	Post Digr.	Ret. Graph	Ret. Digr.	L/T Verb.	L/T N.V.	L/T Total	CTBS Read.	CTBS Arith.
Post Digr	1.0	554*	561*	-197	-231	-230	-230	-222
Ret Gra		1.0	1.004*	-034	-067	-088	-019	073
Ret Digr			1.0	-034	-067	-087	-019	072
L/T Verb				1.0	570*	761*	711*	609*
L/T NV					1.0	834*	513*	533*
L/T Total						1.0	649*	622*
CTBS Read							1.0	630*
CTBS Arith.								1.0

*$p < .05$

**decimals omitted

Table 20**

Product Moment Correlations
Eighth Grade Experimental Group

	Att. Math.	Pre Ach.	Post Ach.	Ret. Ach.	Pre Graph	Pre Digr.	Post Graph	Post Digr.
Att. Math	1.000 (43)	332* (43)	240 (41)	240 (43)	-268 (43)	-275 (43)	167 (42)	177 (42)
Pre Ach		1.000 (43)	330* (41)	314* (43)	-074 (43)	-077 (43)	-108 (42)	-108 (42)
Post Ach			1.000 (41)	891* (41)	049 (41)	039 (41)	-043 (40)	-031 (40)
Ret Ach				1.000 (43)	-042 (43)	-050 (43)	-123 (42)	-112 (42)
Pre Gra					1.000 (43)	999* (43)	221 (42)	239 (42)
Pre Dig						1.000 (43)	225 (42)	242 (42)
Post Gra							1.000 (42)	998* (42)
Post Dig								1.000 (42)

	Ret. Graph	Ret. Digr	MEMM
Att. Math	071 (42)	067 (42)	203 (27)
Pre Ach	-137 (42)	-131 (42)	248 (27)
Post Ach	215 (40)	214 (40)	-032 (26)
Ret Ach	185 (42)	191 (42)	-023 (27)
Pre Gra	316* (42)	340* (42)	328 (27)
Pre Dig	316* (42)	341* (42)	335 (27)
Post Gra	400* (41)	381* (41)	268 (27)

	Ret. Graph	Ret. Digr.	MEMM
Post Dig	391* (41)	374* (41)	278 (27)
Ret. Gra	1.000 (42)	997* (42)	159 (27)
Ret. Dig		1.000 (42)	176 (27)
MEMM			1.000 (27)

(N's in Parenthesis)

*p .05

**decimals omitted

Table 21**

Rank Order Correlations
Eighth Grade Experimental Group

	Att. Math.	Pre Ach.	Post Ach.	Ret. Ach.	Pre Graph	Pre Digr.	Post Graph	Post Digr.
Att. Math	1.000	243*	186	158	-185	-181	114	116
Pre Ach		1.000	236*	285*	-195	-198	-134	-132
Post Ach			1.000	724*	046	051	013	024
Ret Ach				1.000	-029	-036	-075	-080
Pre Gra					1.000	971*	063	068
Pre Dig						1.000	092	097
Post Gra							1.000	971*
Post Dig								1.000

	Ret. Graph	Ret. Digr.	MEMM
Att. Math	064	042	084
Pre Ach	-147	-144	136
Post Ach	129	104	075
Ret Ach	037	027	053
Pre Gra	339*	355*	244
Pre Dig	364*	375*	263
Post Gra	406*	386*	239
Post Dig	397*	372*	245
Ret Gra		969*	163
Ret Dig			175
MEMM			1.000

*$p < .05$
**decimals omitted

Table 22**

Product Moment Correlations
High School Experimental Group

	Att. Math.	Pre Ach.	Post Ach.	Ret. Ach.	Pre Graph	Pre Digr.	Post Graph	Post Digr.
Att Math	1.000 (20)	090 (19)	251 (15)	-090 (11)	039 (20)	065 (20)	319 (17)	302 (17)
Pre Ach		1.000 (19)	088 (15)	108 (10)	074 (19)	086 (19)	151 (17)	186 (17)
Post Ach			1.000 (15)	941* (10)	-099 (15)	-104 (15)	-324 (15)	-324 (15)
Ret Ach				1.000 (11)	-071 (11)	-083 (11)	-104 (10)	-102 (10)
Pre Gra					1.000 (20)	998* (20)	319 (17)	343 (17)
Pre Dig						1.000 (20)	344 (17)	370 (17)
Post Gra							1.000 (17)	998* (17)
Post Dig								1.000 (17)

	Ret. Graph	Ret. Digr.
Att Math	-003 (11)	002 (11)
Pre Ach	310 (10)	314 (10)
Post Ach	-015 (10)	-037 (10)
Ret Ach	-168 (11)	-180 (11)
Pre Gra	618* (11)	623* (11)
Pre Dig	636* (11)	640* (11)
Post Gra	725* (10)	742* (10)
Post Dig	736* (10)	754* (10)
Ret Gra	1.000 (11)	998* (11)
Ret Dig		1.000 (11)

(N's in parenthesis)

*$p < .05$

**decimals omitted

Table 23**

Rank Order Correlations
High School Experimental Group

	Att. Math.	Pre Ach.	Post Ach.	Ret. Ach.	Pre Graph	Pre Digr.	Post Graph	Post Digr.
Att Math	1.000	183	214	-058	-076	-065	212	205
Pre Ach		1.000	031	071	093	080	169	162
Post Ach			1.000	907*	030	010	-226	-237
Ret Ach				1.000	019	038	-023	-023
Pre Gra					1.000	989*	-030	-007
Pre Dig						1.000	-015	007
Post Gra							1.000	952*
Post Dig								1.000

	Ret. Graph	Ret. Digr.
Att Math	056	-038
Pre Ach	345	372
Post Ach	068	046
Ret Ach	-057	0
Pre Gra	273	330
Pre Dig	275	333
Post Gra	511*	539*
Post Dig	511*	539*
Ret Gra	1.000	917*
Ret Dig		1.000

*$p < .05$

**decimals omitted

Comparisons of Content Structure and Cognitive Structure

One way to compare content structure and cognitive structure is to examine the correspondence, or lack of it, between the multidimensional scaling solutions for the digraph similarity matrix (or graph similarity matrix) and the median (or mean) RC matrix. This method allows one to look only at group data.

A second method of comparing content structure with cognitive structure is the Euclidean distance. The Euclidean distance is obtained by squaring each difference between corresponding elements of two matrices (e.g. a S's RC matrix and the digraph similarity matrix), summing the squares, taking the square root of this sum, and dividing by ninety (the number of off-diagonal elements in each matrix). For each S's RC matrix (at each testing time), the Euclidean distances between that RC matrix and the content graph and digraph similarity matrices, respectively, were calculated. These Euclidean distances indicate how well a RC matrix matches one of the content structure matrices.[1] The smaller the distance, the closer the RC matrix comes to matching the content matrix. Descriptive statistics for the Euclidean distances at each testing time are given in Table 24.

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[1] Since the smallest value of a RC is zero, some RC matrices consist only of off-diagonal elements that are zero. This may cause a Euclidean distance to be smaller than it should be, since it is possible to be further away from the content structure (e.g., some Euclidean distances between certain PC matrices and the content matrix are larger than the distance between a "zero" RC matrix and the content matrix).

Table 24

Descriptive Statistics for Euclidean Distances Between
Ss RC Matrices and Content Similarity Matrices

School Level	Treatment	Statistic	Pretest Graph	Pretest Digraph	Posttest Graph	Posttest Digraph	Retention Test Graph	Retention Test Digraph
SIXTH GRADE	Experimental Ss	Sample Size	32	32	31	31	22	22
		Mean	.038	.037	.036	.035	.033	.032
		Standard Deviation	.006	.005	.005	.005	.006	.006
		Range Max	.0404	.0395	.0414	.0405	.0404	.0395
		Range Min	.0172	.0179	.0212	.0219	.0197	.0201
	Control Ss	Sample Size	25	25	23	23	22	22
		Mean	.039	.038	.038	.038	.038	.037
		Standard Deviation	.002	.002	.005	.005	.005	.004
		Range Max	.0404	.0395	.0404	.0395	.0404	.0395
		Range Min	.0318	.0311	.0211	.0207	.0261	.0255

Table 24 (cont.)

School Level	Treatment	Statistic		Pretest		Posttest		Retention Test	
				Graph	Digraph	Graph	Digraph	Graph	Digraph
EIGHTH GRADE	Experimental Ss	Sample Size		43	43	42	42	42	42
		Mean		.039	.038	.031	.030	.030	.029
		Standard Deviation		.002	.002	.005	.005	.006	.006
		Range	Max	.0404	.0395	.0400	.0392	.0404	.0395
			Min	.0329	.0326	.0189	.0190	.0124	.0126
	Control Ss	Sample Size		42	42	40	40	43	43
		Mean		.038	.037	.035	.034	.034	.033
		Standard Deviation		.004	.004	.007	.007	.007	.007
		Range	Max	.0404	.0395	.0404	.0395	.0404	.0395
			Min	.0183	.0182	.0179	.0176	.0137	.0140

Table 24 (cont.)

School Level	Treatment	Statistic	Pretest		Posttest		Retention Test	
			Graph	Digraph	Graph	Digraph	Graph	Digraph
HIGH SCHOOL	Experimental Ss	Sample Size	20	20	17	17	11	11
		Mean	.039	.038	.032	.032	.031	.031
		Standard Deviation	.001	.001	.005	.004	.005	.005
		Range Max	.0404	.0395	.0384	.0377	.0372	.0366
		Range Min	.0352	.0343	.0222	.0222	.0202	.0206
	Control Ss	Sample Size	14	14	11	11	6	6
		Mean	.039	.038	.037	.036	.034	.034
		Standard Deviation	.001	.001	.004	.004	.004	.004
		Range Max	.0404	.0395	.0404	.0395	.0397	.0388
		Range Min	.0353	.0346	.0249	.0255	.0298	.0298

A nonparametric analysis of variance (Bradley, 1968) was performed on the Euclidean distance data at each school level. Since there was a loss of subjects at retention test in the sixth grade and high school samples, retention test data was not included in this analysis. Results showed a significant treatment effect ($p < .05$ at the sixth grade level; $p < .01$ at the eighth grade and high school levels) at all three levels.

For each cell of the 2 x 3 x 3 (treatment by school level by test occasion) design, a Euclidean distance between each mean and median RC matrix and the digraph and graph similarity matrices was calculated. Table 25 presents the results of these calculations. The Euclidean distance between each PC matrix and the content similarity matrices also is presented in this table.

Table 25

Euclidean Distances Between Mean RC, Median RC, and PC Matrices and Content Structure Matrices

Level	Treatment	Content Matrix	Median RC Matrix			Mean RC Matrix			PC Matrix
			Pre	Post	Ret	Pre	Post	Ret	Post
Sixth Grade	Experimental	Digraph	.0395	.0384	.0348	.0351	.0295	.0255	.0669
		Graph	.0404 (32)	.0393 (31)	.0355 (22)	.0359 (32)	.0302 (31)	.0261 (22)	.0763 (31)
	Control	Digraph	.0395	.0395	.0395	.0378	.0364	.0354	*
		Graph	.0404 (25)	.0404 (23)	.0404 (21)	.0387 (25)	.0373 (23)	.0363 (21)	* (24)
Eighth Grade	Experimental	Digraph	.0395	.0314	.0269	.0377	.0238	.0196	.0735
		Graph	.0404 (43)	.0320 (42)	.0272 (42)	.0386 (43)	.0243 (42)	.0199 (42)	.0829 (41)
	Control	Digraph	.0393	.0387	.0385	.0366	.0307	.0286	.1453
		Graph	.0402 (42)	.0396 (40)	.0394 (43)	.0374 (42)	.0315 (40)	.0293 (43)	.1548 (39)
High School	Experimental	Digraph	.0389	.0337	.0328	.0370	.0279	.0254	.0435
		Graph	.0398 (20)	.0343 (17)	.0332 (11)	.0378 (20)	.0285 (17)	.0259 (11)	.0529 (17)
	Control	Digraph	.0395	.0386	.0343	.0379	.0339	.0305	.0859
		Graph	.0404 (14)	.0394 (11)	.0349 (6)	.0388 (14)	.0347 (11)	.0311 (6)	.0953 (11)

*No usable results
(N's in parentheses)

CHAPTER V

Discussion, Future Work, and Summary

A brief summary and discussion of the results of the study are presented in this chapter. The reader is reminded that results of all analyses are presented in Chapter IV and thus the statistical tables will not be repeated here.

Discussion of Results

Content structure. Content structure was mapped with the methods of digraph analysis, graph analysis, and task analysis. Concepts deemed as being most crucial, in a mathematical sense, for understanding the material were: "probability," "independent," "event," "zero," "equally likely," "intersection," "trial," "experiment," "mutually exclusive," and "outcome". These ten concepts were used as the key concepts in the digraph and graph analyses.

The largest element in the digraph distance matrix (see Table 6) is 4, and this distance occurs only twice. The largest element in the graph matrix (see Table 8) is 3. Thus the digraph and graph are "strong" (Harary, et. al., 1965) indicating a tight formal structure in the subject matter, as would be expected. Interrelations among concepts in the two distance matrices were examined with Kruskal's (1964) multidimensional scaling procedure. For the purposes of this study, it was decided to accept the smallest number of dimensions that would allow a "good" fit to the data (see Chapter IV for a discussion of the fit criterion). A two-dimensional space (stress = .041 for the digraph; stress = .005 for the graph) was selected in both analyses. In both cases the scaling solutions (see Figures 3 & 4) match our understanding of the instructional material. In examining Figure 3, for example, one finds that "outcome" is clustered with "experiment" and "event," "probability" is clustered with "intersection." Examining larger clusters we see that "mutually exclusive," "independent," and "intersection"/"probability" are related to "event," "trial" and "equally likely" cluster with "experiment"/"outcome." Finally, "zero" is related most closely to "probability"/"intersection." Thus there appears to be a clustering on the basis of abstractness of the concepts. For example, "experiment" is less abstract than "event." The third cluster which includes "probability" gives us the unifying mathematical concepts. This fits nicely with the idea of probability being an abstract model of the physical world.

Problem solving. It is apparent from the analyses of variance results of achievement data that each experimental group did significantly better ($p < .01$) after instruction as compared to before instruction. At the same time while control groups performed slightly better at posttest and retention test as compared to pretest this difference was not significant. From the mean scores for each test occasion (see Tables 10, 11, & 12), we see that each experimental group was able to solve problems much better after instruction than before and this was not true for the control groups. The probability instructional material, then, produced learning of probability and this learning was due solely to the instructional material (i.e. a significant treatment effect). Additionally, this learning was retained as judged by retention test means (in fact, eighth grade and high school experimental Ss performed better at retention test than at posttest).

The analysis of variance which includes school level effect indicated that older Ss performed significantly better on the achievement test than did younger Ss.

Results of correlation analyses. An examination of the results of the within experimental group correlation analyses leaves us with a somewhat less enthusiastic appraisal of the WA results. The mean/median RC matrices distinguish the experimental and control groups well and in the same manner as the achievement data. However, with the exception of the sixth grade data, we did not obtain a significant correlation between WA data and achievement data within an experimental group.

The sixth grade data indicates that posttest achievement was correlated significantly ($p < .05$) with retention achievement, post and retention WA results, and ability measures (see Tables 18 & 19; note that the rank correlation coefficient was not significant for retention WA data even though the product-moment correlation coefficient was). Retention achievement was not correlated significantly with WA data; ability data was not related significantly to WA data. Attitude toward mathematics was correlated significantly with retention WA data but nothing else. All intercorrelations among post achievement, retention achievement, and ability measures were significant for the sixth grade experimental Ss.

Achievement data and WA data intercorrelations were not significant for the eighth grade experimental group. The ability measure, Minimum Essentials of Modern Mathematics, was not correlated significantly with any other variable. Attitude toward mathematics was correlated significantly only with pretest achievement. All achievement data intercorrelations were significant.

Neither attitude toward mathematics nor achievement data was correlated significantly with WA data within the high school experimental group. Attitude was not related significantly to any other variable. The only achievement data correlation coefficient that was significant was the correlation between post and retention achievements.

Ability and cognitive structure after instruction were correlated significantly in the sixth grade experimental group, but not in the eighth grade experimental group. Since the ability measure in the eighth grade group was quite specific and there was a significant amount of missing data, the author is inclined to accept the results from the sixth grade group. This result agrees with Shavelson's (1970) findings that some ability and WA data are related. However, the reader is cautioned that patterns in the sixth grade correlation analysis did not appear in the other school levels and thus the link between ability and WA data is tenuous at best.

We were not able to demonstrate consistently a significant relationship between cognitive structure after instruction and either achievement, attitude, or ability within the experimental groups. In general, attitude toward mathematics was not related to other variables in the study. However, achievement data and WA data were related strongly in the sixth grade experimental group. At the eighth grade and high school levels, achievement data and WA data were not correlated significantly, but the correlation coefficient was generally in the appropriate direction although near zero. This finding is in agreement with past studies (cf. Johnson, 1967; Shavelson, 1970). It may be that the larger variation present in the sixth grade experimental group was the cause of that being the only group with significant results in the correlation analyses.

Thus it appears that two possible interpretations of the WA test results are possible: a) the WA test provides a measure of group learning but not individual learning; b) the WA test provides a measure of learning and this learning is of a different type than learning to solve problems and thus the WA results may not always agree highly with conventional achievement results (much the same as attitude and achievement are not always highly related). The author prefers the second interpretation, particularly since many of the problems on the achievement test could be solved by an algorithm, even if the student did not understand fully the relationships between various concepts. (A third interpretation, the WA test does not measure learning of mathematical structure, was discounted by the author due to the striking consistency of between treatment groups performance on the WA test and the consistency of this difference with achievement test differences. Also the results of the multidimensional scaling procedure in each experimental group indicate learning of structure took place.)

Cognitive structure. Examining the WA test results (see Figures 6-30) indicates that eighth grade Ss did best in terms of learning content structure but that both high school and eighth grade Ss performed better than sixth grade Ss. Results of the PC test (see Figures 31-35) indicated that eighth grade Ss performed worst with high school Ss performing best. The author suspects that this result on the PC test is due partly to the large number of unusable results at the sixth grade level. That is, only the best achievers in sixth grade were able to respond to the PC test and thus the mean score was inflated.

Thus, in general, there appears to be a strong maturation effect on the test scores. A portion of this effect was probably due to older Ss having higher reading ability and more experience with mathematics. However, some of the effects may be due to older Ss being able to retain or learn better the abstract portion of the mathematics.

The pattern of intercorrelations between WA results at different test times within experimental groups also provide some interesting observations. Posttest WA results always were correlated significantly with retention test WA results. Pretest WA results were correlated significantly with posttest

WA results at the sixth grade level and were correlated significantly with the retention test WA results at the other two school levels. In the case of non-significant results the direction of the correlation coefficient was positive and usually approached significance. The correlations between posttest and retention test WA results further indicate the stability of learning of mathematical structure. We suggest the relationship between before-instruction WA data and after-instruction WA data indicates that Ss were not able to separate totally the common meaning of the key concepts and the specific (mathematical) meaning of the concepts. That is, while the instructional material presents only the mathematical meaning of the key concepts, Ss "add" this extra understanding of concepts to their organization of concepts in memory but also retain the everyday meaning of the concepts. Thus instruction may produce only a change in the quality of concept organization in memory, but not necessarily cause S to reorganize radically his cognitive structure.

Content structure versus cognitive structure. Examining the Euclidean distance scores between the content structure distance matrices and the mean/median RC matrices (see Table 25) one observes the same pattern of learning due to instruction as was found in the achievement data. The difference in learning between experimental and control groups is also apparent in the RC and PC matrices. Thus it was concluded that experimental Ss were not only better at solving problems after instruction, but also learned some mathematical structure (as measured by the WA and PC tests). Note also that experimental Ss' cognitive structures were more like content structure at retention test than at posttest (see Table 25). This demonstration of a lasting change in cognitive structure is important since prior studies have not examined this.

The fact that experimental Ss exhibited the same general pattern, in terms of mean scores, on both achievement tests and cognitive structure tests and the after-instruction scaling solutions indicate that the cognitive structure tests measure learning (of mathematical structure). The learning of mathematical structure was retained.

Formative evaluation. The results of the multidimensional scaling solutions provide the most useful information for formative evaluation. The resultant dimensionality of best fit and clustering of concepts provide a more easily interpretable description of the RC matrices.

Several patterns emerge from the best fit dimensionalities (summarized in Table 26). With the exception of the high school control group's WA retention test result, all control group median RC matrices and all pretest median RC matrices contained too many zero elements for the obtainment of a scaling solution. (The high school control group at retention test had a very small N and these students appeared quite intelligent which probably caused this exception.) This strongly supports the hypothesis that Ss were unfamiliar with probability prior to instruction and that the control group Ss did not learn the mathematical structure of the probability text. In general, the median RC matrices fit well in a smaller dimensionality than the mean RC matrices.

Table 26

Dimensionality of Best Fit for RC Matrices

		Pre		Post		Ret	
		Mean RC	Med. RC	Mean RC	Med. RC	Mean RC	Med. RC
Sixth Grade	Exp.	4	*	3	1	3	1
	Con.	4	*	3	*	3	*
Eighth Grade	Exp.	4	*	3	2	3	3
	Con.	4	*	4	*	3	*
High School	Exp.	4	*	4	1	3	2
	Con.	2	*	4	*	3	2

* indicates inability to calculate a scaling solution due to a majority of the elements in the RC matrix being zero

Following instruction, the experimental groups median RC matrices fit well in two dimensions or less (except for the eighth grade retention test median RC matrix). Posttest and retention test mean RC matrices for experimental groups tend to fit well in three dimensions (except the high school posttest mean RC matrix). Also the mean RC matrices tend to fit in a smaller space following instruction as compared to pretest results especially in the experimental groups.

It appears that relationships between concepts not only approach that presented by text, but also approach the same dimensionality. We conjecture that the third dimension in the mean RC matrices for experimental groups is necessitated by the common meaning associations still present in Ss' memories after instruction.

The summarization of best fit dimensions for the PC matrices is presented in Table 27. No pattern exists for control groups as would be expected. The experimental groups results tend toward three dimensions with the exception of the sixth grade experimental group. The experimental groups PC matrices seem to resemble the mean RC matrices rather than the median RC matrices.

Table 27

Dimensionality of Best Fit for PC Matrices

Sixth Grade	Exp.	4
	Con.	*
Eighth Grade	Exp.	3
	Con.	2
High School	Exp.	3
	Con.	4

The scaling solution indicates how concepts were clustered in Ss' cognitive structures. These clusters can be compared to the text structure and thus provide information as to which relationships Ss learned well and which ones they did not learn well. We suggest that this information may be useful for formative evaluation. For example, examining the post and retention scaling solutions (see Figures 6-30) for experimental Ss, we see that the overall general structure presented by text appears to be present in Ss' memories after instruction, but there appears to be confusion over the distinction between "outcome" and "event." Several scaling solutions show that "mutually exclusive" and "independent" cluster with "outcome" rather than "event." We observe also that, in general, the central concepts ("event," "outcome," "experiment," "trial") are not well distinguished in Ss' memories. Thus Ss have learned the general structure presented by text, but are lacking many refinements in this structure. In terms of formative evaluation, we would suggest more comparing/contrasting of central concepts be presented in the text. Note that this suggestion is quite dissimilar from suggestions that might arise from achievement test results. Achievement test results indicate areas of difficulty in solving problems and formative evaluation suggestions based on achievement tests are often stated in terms of problem solving practice or more explanation of algorithms. Results of the WA test indicate which relationships between concepts need more emphasis regardless of whether the relationship involves problem solving.

The only pattern the author could discern for control group results was a tendency to associate concepts on the basis of familiarity. Here again the clusters indicate the control groups did not learn the probability text structure. The results also indicate that responses to the WA test by experimental Ss were not a random phenomena.

Future Work

Before proceeding with some suggestions for future research, it is necessary to note some inappropriate uses of the cognitive structure instruments. From the experimental groups correlation analyses, we conclude that the WA test is not a substitute for the usual achievement test in terms of individual persons. While we feel that the WA test measures learning of mathematical structure, it is apparent that the WA test is not a good predictor of learning to solve problems. The WA test does appear to be a good predictor of achievement in terms of group means.

Using the present scoring method, the PC test is not appropriate for predicting either the WA test data or problem solving data on an individual basis. Unless an inordinate amount of testing time is used or else a different scoring procedure is used, the PC test should be used only for group distinctions and not individual distinctions. Secondly, it appears that the PC test involves the "ability to write about mathematics" and this ability is not well developed in students, particularly at the elementary school level. This probably is due to lack of experience in writing about mathematics and the investigator feels this skill should be practiced more in the mathematics classroom.

Finally, a simpler scoring procedure would be necessary if individual teachers were to use the WA and PC tests in their classrooms. Even with a computer, the task of putting the results into an interpretable form is quite time consuming.

Two important research questions remain before one could put these procedures to general use. First, do improvements to the text like those suggested above produce a corresponding improvement on the WA and PC tests? This question needs to be answered to assure the usefulness of the cognitive measures in formative evaluation. The second problem of interest is whether a different structure of probability presented by text produces a different cognitive structure in the student. That is, we have shown that the text produces a change in cognitive structure, but we have not shown whether this change is unique with respect to each particular text structure or the same for all texts.

Summary

The purpose of this study was to define mathematical structure operationally and to examine communication of mathematical structure to students by text. Mathematical structure was defined as the relationships between concepts within a set of abstract systems. Content structure was mapped by digraphs, graphs, and task analysis. Cognitive structure in S's memory was measured by the word association technique and the paragraph construction technique. Ss (N=180) participating in the study were from the sixth grade, eighth grade, and high school (grades 9-12) levels. Ss were divided randomly into experimental and control groups at each school level. Experimental Ss read a programmed text concerning elementary probability, while control Ss read a programmed text concerning mathematics not related to probability. The experiment was conducted during regular school hours and, in most cases, in the regular mathematics classroom. The experiment lasted approximately two weeks plus a retention test period. Ss were pretested with attitude toward mathematics, achievement in probability, and WA instruments; posttested with WA, achievement, and PC instruments; and retention tested with WA and achievement instruments. Experimental Ss scored significantly better on the achievement test at post and retention test times as compared to pretest. Experimental Ss scored significantly better at post and retention test times than did control Ss. Digraph analysis and graph analysis provided representation of content structure that was interpretable and agreed with our understanding of the instructional material. The word association and paragraph construction techniques were useful in examining the learning of mathematical structure at all levels. Experimental Ss' cognitive structures resembled the content structure following instruction; this was not true of control Ss. It was concluded that experimental Ss learned how to solve probability problems and learned a significant portion of the text structure as a result of instruction. There was both a treatment group and school level difference on the achievement, W_A, and PC tests. The WA and PC tests appeared to be useful for formative evaluation and gave different information than did the achievement test.

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Appendix A

Learning Measures

ACHIEVEMENT TEST

INSTRUCTIONS

This is a test about probability. You will have all the time you need to finish the test. Read the questions carefully and answer them to the best of your ability.

Please try all the questions. However, if you find some questions too hard, leave them until you have answered the easier ones. Then you may come back and try the harder ones. Try to do the best you can.

The questions are of two types:

1) In the experiment of tossing a fair coin,

P(Heads) = ____________

2) In the experiment of tossing a fair coin, P(Heads) equals

(A) 1/2 (B) 3/4 (C) 1 (D) none of these

In questions like question 1 you are to put your answer in the space provided. In questions like question 2 you are to circle the letter in front of the correct answer. Circle only one answer for each question.

In the example questions above, your answers would look like:

1) In the experiment of tossing a fair coin,

P(Heads) = 1/2

2) In the experiment of tossing a fair coin, P(Heads) equals

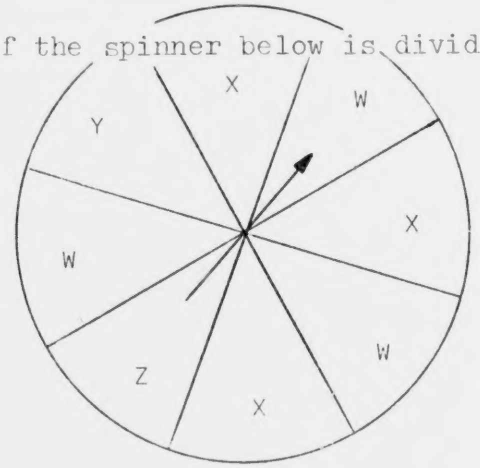
[✓ (A)] 1/2 (B) 3/4 (C) 1 (D) none of these

Are there any questions about what you are supposed to do?

You may turn the page and begin.

1. Suppose you were going to pick a marble out of a bag that was filled with 10 white marbles, 8 green marbles, and 4 red marbles.
   What is the probability of picking a green marble? __________

2. The face of the spinner below is divided into 8 equal areas.



   What is P(X)? ______________

3. A teacher wrote the numbers 1, 2, 3, 5, 7, 8, 10 on the blackboard, and said "I am thinking of one of these numbers." What is the probability that she is thinking of an even number? ______________

4. There are 9 possible outcomes for an experiment. The event "Blue" contains 6 of the possible outcomes. What is P(Blue)? ______________

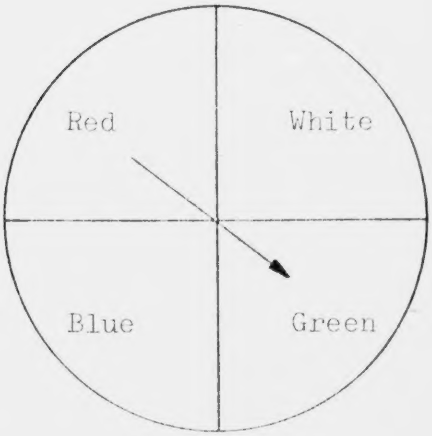
5. A bag contains 15 marbles. P(Blue) = 1/5. How many blue marbles are in the bag? ______________

6. You have a box containing 60 tennis balls. Some of the balls are red and some are white. If you draw a ball from the box (without looking), P(Red) = 1/3. How many white tennis balls are in the bag? ______________

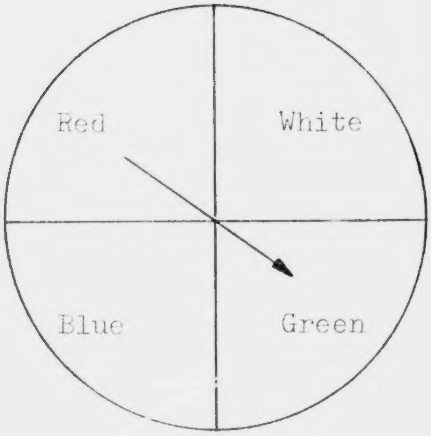
7. If you toss two coins, P(at least one Head) = __________.

8. If you toss three coins, P(at least one Tail) = __________.

Consider the experiment of spinning both the spinners below (Questions 9-14).


Red
White
Blue
Green

Spinner 1


Red
White
Blue
Green

Spinner 2

9. Draw the tree diagram and list all the possible outcomes of the experiment.

Tree Diagram

Outcomes

10. P(at least one Green) = ________________

11. P(exactly one White) = ________________

12. P(at least one White and no Green) = ________________

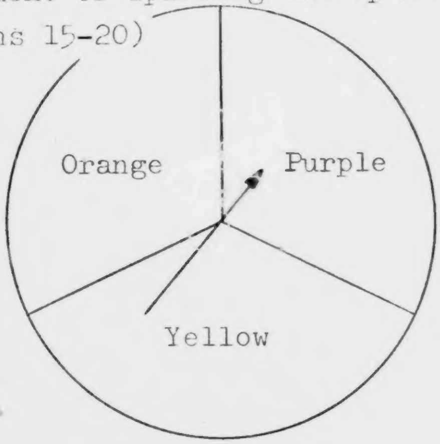
13. List the outcomes in the event "No Blues".

____

14. List the outcomes in the event "at least one Red".

____

Consider the experiment of spinning the spinner below twice in a row. (Questions 15-20)


Orange
Purple
Yellow

15. Draw a tree diagram showing all possible outcomes of the experiment.

Tree Diagram

Outcomes

16. P(different color on each spin) = ______________

17. P(no Purples) = ______________

18. List the outcomes in the event "at least one Purple".

____

19. Are the two events "at least one Orange" and "at least one Yellow" independent? ______________

20. Are the two events "at least one Orange" and "at least one Yellow" mutually exclusive? ______________

Consider the experiment of drawing twice from a bag of marbles. The bag contains two red marbles and three black marbles. (Questions 21-24)

21. If you do not put the marble you picked on the first draw back in the bag before making the second draw, then

P(R,B) = ____________.

22. If you do put the marble you picked on the first draw back in the bag before making the second draw, then

P(B,R) = ____________.

23. If you do not put the marble you picked on the first draw back in the bag before making the second draw, then

P(at least one Red) = ____________.

24. If you do put the marble you picked on the first draw back in the bag before making the second draw,

P(same color on both draws) = __________.

25. What do we mean when we say that two events are independent?

____

____

____

26. What do we mean when we say that two events are mutually exclusive?

____

____

____

27. If you toss a coin twice, P(No Tails) = ______________.

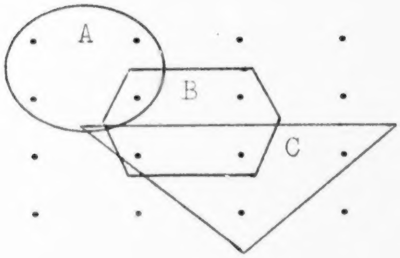
28. If you toss a coin three times, what is the probability of getting three Heads? ______________.

MULTIPLE CHOICE - Circle the correct answer.

29. An event is a set of
    - (A) probabilities
    - (B) experiments
    - (C) outcomes
    - (D) fractions
    - (E) none of these

30. The number of ____________ of the experiment refers to the number of times we repeat the experiment.
    - (A) outcomes
    - (B) events
    - (C) dice
    - (D) trials
    - (E) none of these

Each of the 16 dots represents a possible outcome of an experiment. Assume the outcomes are equally likely. (Questions 31-35)



31. A pair of events that is independent is

(A) A,B (B) B,C (C) A,C (D) A,A (E) None of these

32. P(A) =

(A) 4 (B) 1/2 (C) 1/4 (D) 3/16 (E) None of these

33. P(B and C) =

(A) 3/8 (B) 1/4 (C) 2/6 (D) 2/16 (E) None of these

34. A pair of events that is mutually exclusive is

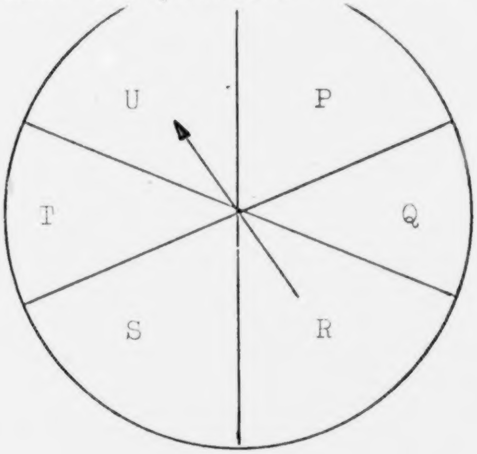
(A) A,B (B) B,C (C) A,C (D) A,A (E) None of these

35. P(not-A) =

(A) 1/4 (B) 3/4 (C) 1/2 (D) 3/8 (E) None of these

Test 1

1. List the possible outcomes of spinning the spinner below.

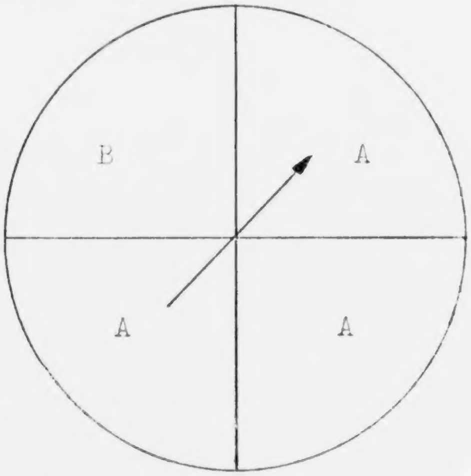


____

2. A spinner has the numbers 5 through 15 printed on it. List the possible outcomes in the event "a number less than 12".

____

3. How many possible outcomes are there for the experiment of spinning the spinner below?



(A) 0 (B) 1 (C) 2 (D) 4 (E) none of these

4. If the face of a spinner is all blue P(Red) =

(A) 1 (B) 1/2 (C) 0 (D) uncertain

5. If the probability of Red on a spinner is equal to 1, which of the following must be true?

(A) Spinner might be blue (B) Spinner could have two colors.

(C) Spinner is all red. (D) Not sure of the color.

6. A bag has 6 marbles in it. Some are red and some are black. How many black marbles must be in the bag if P(Red) = P(Black)?

(A) 2 (B) 3 (C) 4 (D) 6 (E) can't tell

7. A bag contains 5 red and 6 green marbles. You pull one marble out. After that, the probability of pulling out a red marble is 2/5. The color of the marble you pulled out is:

(A) red (B) green (C) could be either red or green

(D) don't know

8. In the experiment of throwing a fair die, P(number greater than 3) = ________________.

9. A bag contains eleven marbles. The marbles are numbered 1 to 11. In the experiment of drawing a marble from the bag, what is the probability of picking a marble with an even number? ________________.

10. A bag contains eight marbles. Some are red and some are black. P(Red) = 1/4. How many black marbles are in the bag?

(A) 6 (B) 4 (C) 2 (D) 1 (E) none of these

Test 2

Think of the experiment of tossing two dice, a red one and a white one.

1. Which of the following are the same outcomes of the experiment?

I.	II.	III.	IV.
(6,2)	(2,6)	(5,1)	(2,6)

(A) I and II

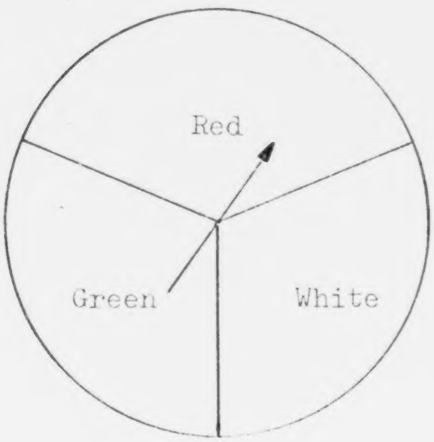
(B) II and IV

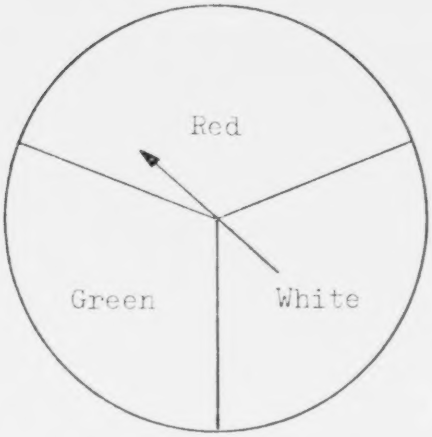
(C) I, II, and IV

(D) I, II, III, and IV

(E) None of the outcomes are the same.

Consider the experiment of spinning both the spinners below (Qeustions 2-6).


Red
Green
White


Red
Green
White

2. Draw the tree diagram and list all the possible outcomes of the experiment.

3. P(R,W) = ______________.

4. P(at least one white) = ______________.

5. P(exactly one red) = ______________.

6. List the outcomes in the event "No Greens".

____

7. If you toss three coins, P(exactly one Head) = ___________.

8. If you toss three coins, P(at least two Heads) =

____

9. A bag contains one blue and one white marble. One marble is removed from the bag and a coin is tossed. What is P(W,T)?

(A) 1/4

(B) 3/4

(C) 2/2

(D) 1/2

(E) 0

10. A bag contains 3 green marbles, 2 blue marbles, and 1 black marble. Joe draws 1 marble and gets a black one. He does not put the black marble back in the bag. What is the probability that the next marble he draws will be green?

(A) 3/6

(B) 3/5

(C) 2/6

(D) 2/5

(E) 0

Random Orders of Key Concepts for Word Association Test

1

intersection
outcome
mutually exclusive
trial
event
experiment
independent
equally likely
probability
zero

2

independent
trial
probability
equally likely
mutually exclusive
outcome
event
intersection
zero
experiment

3

probability
intersection
independent
equally likely
trial
experiment
event
outcome
mutually exclusive
zero

4

probability
independent
trial
intersection
mutually exclusive
experiment
equally likely
zero
outcome
event

Random Orders for Concept-Pairs for Paragraph Construction Tests

PC Test 1

1

experiment - zero
outcome - independent
probability - event
trial - intersection
equally likely - mutually exclusive

2

outcome - independent
trial - intersection
experiment - zero
probability - event
equally likely - mutually exclusive

3

equally likely - mutually exclusive
probability - event
trial - intersection
experiment - zero
outcome - independent

4

equally likely - mutually exclusive
experiment - zero
outcome - independent
probability - event
trial - intersection

PC Test 2

1

zero - equally likely
trial - independent
outcome - mutually exclusive
event - intersection
probability - experiment

2

probability - experiment
trial - independent
event - intersection
zero - equally likely
outcome - mutually exclusive

3

zero - equally likely
event - intersection
probability - experiment
trial - independent
event - intersection

4

outcome - mutually exclusive
event - intersection
trial - independent
probability - experiment
zero - equally likely

Appendix B

Numerical Results of Multidimensional Scaling Solutions

Multidimensional Scaling Solution for Digraph Similarity Matrix

	Final Configuration 2 Dimensions Stress = .041		Sorted Configurations* Dimension 1	2
Prb.	0.009,	-0.768	Ind.	Zero
Ind.	-1.243,	0.113	M.E.	Prb.
Event	-0.261,	0.176	Zero	Int.
Zero	-0.474,	-1.411	Event	Exp.
E.L.	1.355,	0.538	Int.	Ind.
Int.	-0.144,	-0.527	Prb.	Event
Tr.	0.454,	0.928	Tr.	Out.
Exp.	0.671	-0.055	Out.	E.L.
M.E.	-0.833,	0.797	Exp.	M.E.
Out.	0.466,	0.209	E.L.	Tr.

*These are the resultant orderings (from negative to positive) of the key concepts along each dimension of the multidimensional scaling solution.

Multidimensional Scaling Solution for Graph Similarity Matrix

	Final Configuration 2 Dimensions Stress = .005		Sorted Configurations Dimension	
			1	2
Prb.	0.391,	-0.486	E.L.	Zero
Ind.	-0.182,	1.314	Out.	Exp.
Event	-0.022,	0.486	Tr.	Prb.
Zero	0.942,	-1.265	Exp.	E.L.
E.L.	-1.328,	-0.464	Ind.	Int.
Int.	0.506,	-0.148	Event.	Out.
Tr.	-0.460,	0.281	Prb.	Tr.
Exp.	-0.326,	-0.759	Int.	Event
M.E.	0.963,	0.831	Zero	M.E.
Out.	-0.484,	0.209	M.E.	Ind.

Multidimensional Scaling Solution for Sixth Grade
Experimental Subjects Pretest Mean
Relatedness Coefficient Matrix

	Final Configuration 4 Dimensions Stress = .010	Sorted Configurations Dimension			
		1	2	3	4
Prb.	-0.303, -0.514, -0.354, -0.353	E.L.	Zero	E.L.	E.L.
Ind.	0.153, 0.434, -0.405, -0.080	Event	E.L.	Ind.	Prb.
Event	-0.324, 0.262, 0.264, -0.200	Prb.	Prb.	Prb.	Exp.
Zero	-0.162, -0.781, 0.236, 0.169	Exp.	Out.	M.E.	Int.
E.L.	-0.766, -0.625, -1.013, -0.701	Zero	Exp.	Int.	Event
Int.	1.080, 0.233, 0.148, -0.290	M.E.	Tr.	Zero	Tr.
Tr.	-0.058, 0.179, 0.381, -0.102	Tr.	Int.	Event	Ind.
Exp.	-0.253, 0.155, 0.556, -0.293	Ind.	Event	Tr.	Zero
M.E.	-0.145, 0.872, -0.325, 0.904	Out.	Ind.	Out.	M.E.
Out.	0.777, -0.215, 0.512, 0.946	Int.	M.E.	Exp.	Out.

Multidimensional Scaling Solution for Sixth Grade
Experimental Subjects Posttest Mean
Relatedness Coefficient Matrix

	Final Configuration 3 Dimensions Stress = .038			Sorted Configurations Dimension 1	2	3
Prb.	0.258,	0.102,	0.036	Zero	E.L.	Int.
Ind.	-0.122,	0.979,	-0.629	Out.	M.E.	M.E.
Event	-0.223,	-0.367,	0.487	Exp.	Event	Ind.
Zero	-0.894,	-0.158,	-0.590	Event	Zero	Zero
E.L.	0.810,	-0.973,	0.213	Ind.	Exp.	Prb.
Int.	0.759,	0.327,	-0.978	Tr.	Prb.	E.L.
Tr.	0.075,	0.555,	0.714	M.E.	Out.	Out.
Exp.	-0.270,	0.013,	1.056	Prb.	Int.	Event
M.E.	0.193,	-0.781,	-0.670	Int.	Tr.	Tr.
Out.	-0.585,	0.303,	0.360	E.L.	Ind.	Exp.

Multidimensional Scaling Solution for Sixth Grade
Experimental Subjects Posttest Median
Relatedness Coefficient Matrix

Final Configuration 1 Dimension Stress = 0.0		Sorted Configuration Dimension 1
Prb.	2.435	Out.
Ind.	-0.039	Event
Event	-0.595	Zero
Zero	-0.039	E.L.
E.L.	-0.039	Tr.
Int.	0.298	Exp.
Tr.	-0.039	M.E.
Exp.	-0.039	Ind.
M.E.	-0.039	Int.
Out.	-1.902	Prb.

Multidimensional Scaling Solution for Sixth Grade
Experimental Subjects Retention Test Mean
Relatedness Coefficient Matrix

	Final Configuration 3 Dimensions Stress = .056			Sorted Configurations Dimension		
				1	2	3
Prb.	-0.386,	-0.061,	-0.150	E.L.	Zero	M.E.
Ind.	-0.468,	0.873,	-0.461	Ind.	Out.	Zero
Event	-0.161,	-0.116,	0.731	Prb.	Int.	Ind.
Zero	0.670,	-0.863,	-0.912	M.E.	Event	Prb.
E.L.	-1.130,	0.165,	0.399	Event	Prb.	Int.
Int.	1.530,	-0.317,	0.104	Out.	E.L.	Out.
Tr.	0.257,	0.181,	0.512	Exp.	Tr.	E.L.
Exp.	0.135,	0.342,	0.574	Tr.	M.E.	Tr.
M.E.	-0.330,	0.264,	-0.929	Zero	Exp.	Exp.
Out.	-0.116,	-0.469,	0.130	Int.	Ind.	Event

Multidimensional Scaling Solution for Sixth Grade Experimental Subjects Retention Test Median Relatedness Coefficient Matrix

	Final Configuration 1 Dimension Stress = .009	Sorted Configuration Dimension 1
Prb.	0.766	E.L.
Ind.	-0.030	Zero
Event	0.730	Ind.
Zero	-1.845	M.E.
E.L.	-1.969	Int.
Int.	0.230	Tr.
Tr.	0.685	Exp.
Exp.	0.705	Event
M.E.	-0.030	Out.
Out.	0.759	Prb.

Multidimensional Scaling Solution for Sixth Grade Control Subjects Pretest Mean Relatedness Coefficient Matrix

	Final Configuration 4 Dimensions Stress = .012	Sorted Configurations Dimension			
		1	2	3	4
Prb.	-0.374, -0.212, -0.785, 0.147	Tr.	Int.	Prb.	E.L.
Ind.	0.393, 0.562, -0.679, 0.051	Zero	Zero	Ind.	Out.
Event	0.418, -0.344, 1.173, -0.004	Prb.	Event	Int.	Zero
Zero	-0.443, -0.479, -0.226, -0.119	Exp.	E.L.	Zero	Event
E.L.	-0.222, -0.280, -0.159, -0.745	E.L.	Prb.	E.L.	Exp.
Int.	0.795, -0.711, -0.612, 0.373	Out.	M.E.	M.E.	Ind.
Tr.	-0.839, 0.800, 0.432, 0.159	Ind.	Out.	Out.	Prb.
Exp.	-0.343, 0.426, 0.790, 0.010	M.E.	Exp.	Tr.	Tr.
M.E.	0.416, 0.031, -0.019, 0.761	Event	Ind.	Exp.	Int.
Out.	0.199, 0.207, 0.086, -0.635	Int.	Tr.	Event	M.E.

Multidimensional Scaling Solution for Sixth Grade Control Subjects Posttest Mean Relatedness Coefficient Matrix

	Final Configuration 3 Dimensions Stress = .053			Sorted Configurations Dimension		
				1	2	3
Prb.	0.556,	-0.285,	-0.731	E.L.	Zero	E.L.
Ind.	-0.016,	0.928,	0.183	Exp.	Out.	Prb.
Event	-0.159,	-0.364,	1.339	Out.	Event	Zero
Zero	0.080,	-1.077,	-0.369	M.E.	Prb.	M.E.
E.L.	-0.827,	0.154,	-0.856	Event	E.L.	Tr.
Int.	0.644,	0.396,	-0.167	Ind.	Exp.	Int.
Tr.	0.683,	0.547,	-0.224	Zero	M.E.	Ind.
Exp.	-0.506,	0.200,	0.637	Prb.	Int.	Out.
M.E.	-0.216,	0.348,	-0.359	Int.	Tr.	Exp.
Out.	-0.239,	-0.847,	0.546	Tr.	Ind.	Event

Multidimensional Scaling Solution for Sixth Grade Control Subjects Retention Test Mean Relatedness Coefficient Matrix

	Final Configuration 3 Dimensions Stress = .044			Sorted Configurations Dimension		
				1	2	3
Prb.	-0.877,	0.599,	-0.412	Prb.	E.L.	Zero
Ind.	-0.359,	1.023,	0.116	Out.	Zero	Prb.
Event	-0.515,	-0.327,	0.634	Event	Event	Int.
Zero	-0.350,	-0.599,	-0.826	Ind.	Out.	E.L.
E.L.	0.657,	-1.459,	-0.205	Zero	M.E.	M.E.
Int.	0.806,	0.359,	-0.382	Exp.	Exp.	Ind.
Tr.	0.454,	0.701,	0.187	Tr.	Int.	Tr.
Exp.	0.085,	0.111,	0.353	M.E.	Prb.	Exp.
M.E.	0.622,	-0.086,	-0.090	E.L.	Tr.	Out.
Out.	-0.523,	-0.323,	0.626	Int.	Ind.	Event

Multidimensional Scaling Solution for Eighth Grade
Experimental Subjects Pretest Mean
Relatedness Coefficient Matrix

	Final Configuration 4 Dimensions Stress = .028	Sorted Configurations Dimension			
		1	2	3	4
Prb.	-0.225, -0.214, -1.028, -0.440	Ind.	Zero	Prb.	Out.
Ind.	-0.936, 0.244, 0.061, -0.083	Prb.	Out.	E.L.	Prb.
Event	0.132, 0.018, 0.880, -0.047	M.E.	E.L.	M.E.	Tr.
Zero	-0.130, -0.948, 0.005, 0.085	Zero	Prb.	Out.	Ind.
E.L.	-0.089, -0.302, -0.747, 0.435	E.L.	Int.	Zero	Event
Int.	0.534, 0.016, 0.777, 0.260	Event	Event	Ind.	Exp.
Tr.	0.166, 0.961, 0.472, -0.372	Tr.	Ind.	Exp.	Zero
Exp.	0.515, 0.363, 0.240, 0.034	Out.	Exp.	Tr.	Int.
M.E.	-0.206, 0.416, -0.439, 0.955	Exp.	M.E.	Int.	E.L.
Out.	0.238, -0.554, -0.221, -0.827	Int.	Tr.	Event	M.E.

Multidimensional Scaling Solution for Eighth Grade
Experimental Subjects Posttest Mean
Relatedness Coefficient Matrix

	Final Configuration 3 Dimensions Stress = .051			Sorted Configurations Dimension 1	2	3
Prb.	0.108,	-0.118,	0.043	Tr.	Zero	E.L.
Ind.	0.521,	0.829,	0.087	Out.	Out.	M.E.
Event	-0.301,	0.113,	0.153	E.L.	Prb.	Int.
Zero	-0.006,	-1.562,	0.095	Exp.	Int.	Prb.
E.L.	-0.544,	0.484,	-0.774	Event	Exp.	Ind.
Int.	1.634,	-0.115,	-0.194	Zero	Event	Zero
Tr.	-0.643,	0.467,	0.422	Prb.	M.E.	Event
Exp.	-0.495,	-0.004,	0.563	M.E.	Tr.	Out.
M.E.	0.339,	0.239,	-0.684	Ind.	E.L.	Tr.
Out.	-0.613,	-0.333,	0.290	Int.	Ind.	Exp.

Multidimensional Scaling Solution for Eighth Grade Experimental Subjects Posttest Median Relatedness Coefficient Matrix

	Final Configuration 2 Dimensions	Stress = .010	Sorted Configurations Dimension 1	2
Prb.	0.134,	-0.118	Ind.	E.L.
Ind.	-1.228,	0.328	M.E.	M.E.
Event	0.135,	-0.025	E.L.	Prb.
Zero	0.104,	1.280	Zero	Event
E.L.	-0.161,	-1.383	Exp.	Out.
Int.	1.702,	0.299	Tr.	Exp.
Tr.	0.132,	0.052	Prb.	Tr.
Exp.	0.129,	0.005	Event	Int.
M.E.	-1.208,	-0.442	Out.	Ind.
Out.	0.263,	0.003	Int.	Zero

Multidimensional Scaling Solution for Eighth Grade Experimental Subjects Retention Test Mean Relatedness Coefficient Matrix

	Final Configuration 3 Dimensions Stress = .051			Sorted Configurations Dimension		
				1	2	3
Prb.	0.042,	-0.123,	0.172	Zero	Zero	M.E.
Ind.	-0.251,	-1.040,	-0.726	Out.	E.L.	Ind.
Event	-0.187,	0.118,	0.472	Ind.	Prb.	E.L.
Zero	-1.261,	-0.965,	0.247	Event	M.E.	Int.
E.L.	0.047,	-0.627,	-0.711	Exp.	Out.	Prb.
Int.	1.640,	0.209,	0.111	Prb.	Event	Zero
Tr.	0.043,	0.150,	0.544	Tr.	Exp.	Out.
Exp.	-0.058,	0.150,	0.416	E.L.	Tr.	Exp.
M.E.	0.381,	0.021,	-0.904	M.E.	Int.	Event
Out.	-0.395,	0.026,	0.379	Int.	Ind.	Tr.

Multidimensional Scaling Solution for Eighth Grade Experimental Subjects Retention Test Median Relatedness Coefficient Matrix

	Final Configuration 3 Dimensions Stress = .043			Sorted Configurations Dimension		
				1	2	3
Prb.	0.108,	-0.077,	0.257	Ind.	Zero	Int.
Ind.	-0.772,	0.653,	-0.855	Tr.	Int.	Ind.
Event	-0.319,	0.041,	0.403	Exp.	Prb.	E.L.
Zero	0.052,	-1.311,	0.672	Event	E.L.	M.E.
E.L.	0.494,	-0.012,	-0.762	Out.	Out.	Prb.
Int.	1.069,	-0.577,	-0.912	Zero	Exp.	Event
Tr.	-0.361,	0.044,	0.495	Prb.	Event	Out.
Exp.	-0.330,	0.039,	0.446	M.E.	Tr.	Exp.
M.E.	0.379,	1.211,	-0.153	E.L.	Ind.	Tr.
Out.	-0.319,	-0.010,	0.410	Int.	M.E.	Zero

Multidimensional Scaling Solution for Eighth Grade Control Subjects Pretest Mean Relatedness Coefficient Matrix

	Final Configuration 4 Dimensions Stress = .027	Sorted Configurations Dimension			
		1	2	3	4
Prb.	0.505, 0.364, -0.088, -0.330	Ind.	Out.	Zero	Tr.
Ind.	-0.772, 0.778, -0.548, 0.052	E.L.	E.L.	Ind.	E.L.
Event	0.282, -0.327, 0.623, 0.089	Zero	Zero	E.L.	Exp.
Zero	-0.218, -0.511, -0.716, 0.565	Out.	Int.	Prb.	Prb.
E.L.	-0.513, -0.539, -0.505, -0.573	Exp.	Event	Int.	Int.
Int.	0.938, -0.371, -0.039, -0.160	M.E.	M.E.	M.E.	Ind.
Tr.	0.071, 0.783, 0.342, -0.812	Tr.	Exp.	Out.	Event
Exp.	-0.121, 0.343, 0.628, -0.393	Event	Prb.	Tr.	Zero
M.E.	0.013, 0.230, 0.089, 0.988	Prb.	Ind.	Event	Out.
Out.	-0.185, -0.750, 0.214, 0.574	Int.	Tr.	Exp.	M.E.

Multidimensional Scaling Solution for Eighth Grade Control Subjects Posttest Mean Relatedness Coefficient Matrix

	Final Configuration 4 Dimensions Stress = .022				Sorted Configurations Dimension			
					1	2	3	4
Prb.	0.098,	-0.126,	0.144,	0.110	M.E.	E.L.	Zero	Out.
Ind.	-0.553,	0.833,	-0.507,	0.355	E.L.	Zero	Ind.	Event
Event	-0.193,	-0.186,	0.631,	-0.533	Ind.	Event	Out.	Exp.
Zero	-0.048,	-0.301,	-1.069,	0.412	Out.	Prb.	E.L.	Int.
E.L.	-0.568,	-1.090,	-0.098,	0.166	Event	Out.	Int.	Tr.
Int.	1.483,	0.158,	-0.039,	-0.017	Zero	Tr.	M.E.	Prb.
Tr.	0.434,	0.043,	0.816,	-0.003	Prb.	Int.	Prb.	E.L.
Exp.	0.236,	0.487,	0.244,	-0.450	Exp.	M.E.	Exp.	Ind.
M.E.	-0.600,	0.204,	0.075,	0.566	Tr.	Exp.	Event	Zero
Out.	-0.290,	-0.021,	-0.198,	-0.605	Int.	Ind.	Tr.	M.E.

Multidimensional Scaling Solution for Eighth Grade Control Subjects Retention Test Mean Relatedness Coefficient Matrix

	Final Configuration 3 Dimensions Stress = .040			Sorted Configurations Dimension		
				1	2	3
Prb.	0.349,	0.073,	-0.016	M.E.	Zero	E.L.
Ind.	-0.586,	-0.565,	0.068	Ind.	Int.	Zero
Event	0.147,	0.068,	0.721	Zero	Ind.	M.E.
Zero	-0.474,	-1.039,	-0.853	Out.	Event	Int.
E.L.	0.096,	0.572,	-0.981	Tr.	Prb.	Prb.
Int.	1.299,	-0.664,	-0.160	E.L.	M.E.	Ind.
Tr.	-0.018,	0.530,	0.319	Event	Out.	Tr.
Exp.	0.234,	0.602,	0.942	Exp.	Tr.	Out.
M.E.	-0.662,	0.111,	-0.649	Prb.	E.L.	Event
Out.	-0.385,	0.311,	0.610	Int.	Exp.	Exp.

Multidimensional Scaling Solution for High School Experimental Subjects Pretest Mean Relatedness Coefficient Matrix

	Final Configuration 4 Dimensions Stress = .031				Sorted Configurations Dimension			
					1	2	3	4
Prb.	-0.387,	-0.586,	-0.036,	-0.856	Zero	Event	Ind.	Prb.
Ind.	-0.141,	0.664,	-0.932,	0.142	Prb.	Prb.	M.E.	E.L.
Event	0.644,	-0.674,	0.547,	0.000	Exp.	Zero	Int.	Tr.
Zero	-0.511,	-0.540,	0.022,	0.849	E.L.	E.L.	E.L.	Event
E.L.	-0.245,	-0.540,	-0.573,	-0.544	Ind.	Out.	Prb.	Exp.
Int.	0.438,	0.556,	-0.576,	0.083	M.E.	M.E.	Zero	Int.
Tr.	0.146,	0.372,	0.663,	-0.449	Tr.	Tr.	Event	Out.
Exp.	-0.274,	0.460,	0.720,	0.023	Out.	Exp.	Tr.	Ind.
M.E.	-0.009,	0.277,	-0.598,	0.645	Int.	Int.	Exp.	M.E.
Out.	0.341,	0.011,	0.763,	0.107	Event	Ind.	Out.	Zero

Multidimensional Scaling Solution for High School Experimental Subjects Posttest Mean Relatedness Coefficient Matrix

	Final Configuration 4 Dimensions Stress = .027	Sorted Configurations Dimension			
		1	2	3	4
Prb.	0.077, -0.363, 0.112, -0.048	Zero	E.L.	Ind.	Ind.
Ind.	-0.275, 0.472, -0.797, -0.697	Out.	Zero	M.E.	Int.
Event	-0.363, -0.054, 0.543, -0.347	Event	Prb.	E.L.	Event
Zero	-1.020, -0.534, -0.120, 0.649	Ind.	Event	Int.	Exp.
E.L.	0.230, -0.759, -0.536, 0.198	M.E.	Int.	Zero	Prb.
Int.	1.294, 0.137, -0.274, -0.686	Prb.	Exp.	Prb.	Out.
Tr.	0.131, 0.370, 0.659, 0.138	Tr.	Out.	Out.	Tr.
Exp.	0.067, 0.151, 0.725, -0.147	E.L.	M.E.	Event	E.L.
M.E.	0.307, 0.363, -0.611, 0.896	M.E.	Tr.	Tr.	Zero
Out.	-0.449, 0.217, 0.297, 0.044	Int.	Ind.	Exp.	M.E.

Multidimensional Scaling Solution for High School Experimental Subjects Posttest Median Relatedness Coefficient Matrix

	Final Configuration 1 Dimension Stress = .012	Sorted Configuration Dimension 1
Prb.	0.675	Zero
Ind.	-1.948	Ind.
Event	0.588	M.E.
Zero	-1.949	Int.
E.L.	0.757	Out.
Int.	0.164	Exp.
Tr.	0.608	Event
Exp.	0.581	Tr.
M.E.	-0.022	Prb.
Out.	0.546	E.L.

Multidimensional Scaling Solution for High School Experimental Subjects Retention Test Mean Relatedness Coefficient Matrix

	Final Configuration 3 Dimensions Stress = .006			Sorted Configurations Dimension		
				1	2	3
Prb.	-0.321,	-0.140,	0.152	Ind.	Zero	Ind.
Ind.	-0.571,	0.568,	-0.994	Event	Prb.	Int.
Event	-0.322,	-0.131,	0.161	Prb.	Exp.	Zero
Zero	0.189,	-1.379,	-0.100	Tr.	Event	Prb.
E.L.	-0.144,	0.785,	0.304	Out.	Tr.	Tr.
Int.	1.598,	0.140,	-0.954	Exp.	Out.	Out.
Tr.	-0.320,	-0.128,	0.157	E.L.	Int.	Exp.
Exp.	-0.315,	-0.132,	0.160	Zero	M.E.	Event
M.E.	0.523,	0.544,	0.955	M.E.	Ind.	E.L.
Out.	-0.317,	-0.127,	0.160	Int.	E.L.	M.E.

B

Multidimensional Scaling Solution for High School
Experimental Subjects Retention Test Median
Relatedness Coefficient Matrix

	Final Configuration 2 Dimensions Stress = .009		Sorted Configurations Dimension	
			1	2
Prb.	1.021,	-0.703	Zero	E.L.
Ind.	-0.004,	0.681	E.L.	Zero
Event	0.006,	0.643	Exp.	Prb.
Zero	-1.579,	-0.944	Tr.	Int.
E.L.	-0.464,	-1.454	Ind.	Tr.
Int.	0.714,	-0.121	Event	Exp.
Tr.	-0.178,	0.286	M.E.	M.E.
Exp.	-0.231,	0.363	Out.	Out.
M.E.	0.278,	0.616	Int.	Event
Out.	0.437	0.633	Prb.	Ind.

Multidimensional Scaling Solution for High School Control Subjects Pretest Mean Relatedness Coefficient Matrix

	Final Configuration 2 Dimensions Stress = .025		Sorted Configuration Dimension	
			1	2
Prb.	-0.670,	0.316	Event	Zero
Ind.	-0.147,	0.022	Prb.	E.L.
Event	-0.670,	-0.761	E.L.	Event
Zero	-0.628,	-1.027	Zero	Int.
E.L.	-0.647,	-0.819	Ind.	Out.
Int.	1.352,	-0.474	Tr.	Ind.
Tr.	0.107,	1.165	Exp.	Prb.
Exp.	0.127,	1.118	Out.	M.E.
M.E.	0.880,	0.551	M.E.	Exp.
Out.	0.297,	-0.091	Int.	Tr.

Multidimensional Scaling Solution for High School Control Subjects Posttest Mean Relatedness Coefficient Matrix

	Final Configuration 4 Dimensions Stress = .025	Sorted Configurations Dimension			
		1	2	3	4
Prb.	-0.114, -0.316, -0.006, -0.584	M.E.	E.L.	E.L.	E.L.
Ind.	-0.519, 0.967, -0.138, 0.184	Ind.	Out.	M.E.	Prb.
Event	-0.429, -0.036, 0.718, -0.281	Event	Zero	Int.	Exp.
Zero	0.219, -0.410, -0.057, 1.283	E.L.	Prb.	Ind.	Event
E.L.	-0.329, -0.581, -0.740, -0.651	Prb.	Event	Zero	Tr.
Int.	1.252, 0.328, -0.250, 0.309	Out.	Tr.	Prb.	Out.
Tr.	0.368, 0.011, 0.290, -0.257	Zero	Exp.	Tr.	Ind.
Exp.	0.237, 0.108, 0.417, -0.434	Exp.	Int.	Exp.	Int.
M.E.	-0.605, 0.392, -0.721, 0.426	Tr.	M.E.	Out.	M.E.
Out.	-0.079, -0.463, 0.486, 0.005	Int.	Ind.	Event	Zero

Multidimensional Scaling Solution for High School Control Subjects Retention Test Mean Relatedness Coefficient Matrix

	Final Configuration 3 Dimensions Stress = .037			Sorted Configurations Dimension		
				1	2	3
Prb.	-0.249,	-0.249,	0.531	E.L.	Zero	Int.
Ind.	0.768,	1.035,	-0.134	Event	Out.	E.L.
Event	-0.435,	-0.118,	0.578	Out.	Prb.	M.E.
Zero	1.031,	-0.815,	-0.378	Tr.	Int.	Zero
E.L.	-0.585,	0.039,	-0.909	Prb.	Event	Ind.
Int.	0.109,	-0.149,	-1.210	Exp.	Tr.	Prb.
Tr.	-0.252,	-0.066,	0.773	M.E.	Exp.	Event
Exp.	-0.160,	-0.060,	0.654	Int.	E.L.	Exp.
M.E.	0.089,	0.660,	-0.691	Ind.	M.E.	Tr.
Out.	-0.315,	-0.277,	0.786	Zero	Ind.	Out.

Multidimensional Scaling Solution for High School Control Subjects Retention Test Median Relatedness Coefficient Matrix

	Final Configuration 2 Dimensions	Stress = .010	Sorted Configurations Dimension 1	 2
Prb.	0.732,	0.331	Zero	E.L.
Ind.	-0.304,	0.373	Ind.	M.E.
Event	-0.250,	0.368	Event	Zero
Zero	-1.318,	-0.928	Out.	Out.
E.L.	0.975,	-1.489	Tr.	Prb.
Int.	0.294,	0.717	M.E.	Event
Tr.	-0.052,	0.928	Exp.	Ind.
Exp.	0.099,	0.851	Int.	Int.
M.E.	0.039,	-0.931	Prb.	Exp.
Out.	-0.213,	-0.221	E.L.	Tr.

Multidimensional Scaling Solution for Paragraph
Construction Data Sixth Grade Experimental Subjects

	Final Configuration 4 Dimensions Stress = .031				Sorted Configurations Dimension			
					1	2	3	4
Prb.	0.243,	-0.251,	0.791,	-0.389	Zero	Event	E.L.	Tr.
Ind.	-0.473,	0.932,	-0.099,	0.409	Ind.	Zero	Int.	Int.
Event	0.089,	-0.709,	0.563,	-0.063	Exp.	E.L.	Zero	Prb.
Zero	-0.941,	-0.572,	-0.529,	0.037	Out.	Prb.	M.E.	Event
E.L.	-0.046,	-0.515,	-0.936,	-0.023	E.L.	M.E.	Ind.	E.L.
Int.	0.588,	0.030,	-0.627,	-0.674	Event	Int.	Tr.	Exp.
Tr.	0.400,	0.495,	0.223,	-0.688	Prb.	Out.	Out.	Zero
Exp.	-0.185,	0.447,	0.545,	0.011	Tr.	Exp.	Exp.	Out.
M.E.	0.468,	0.017,	-0.221,	1.118	M.E.	Tr.	Event	Ind.
Out.	-0.143,	0.126,	0.291,	0.261	Int.	Ind.	Prb.	M.E.

Multidimensional Scaling Solution for Paragraph
Construction Data Eighth Grade Experimental Subjects

	Final Configuration 3 Dimensions Stress = .060			Sorted Configurations Dimension		
				1	2	3
Prb.	0.490,	-1.032,	0.367	E.L.	Prb.	M.E.
Ind.	-0.102,	0.662,	-0.599	Zero	M.E.	E.L.
Event	0.011,	-0.131,	0.304	Out.	E.L.	Ind.
Zero	-0.740,	-0.335,	0.174	Ind.	Zero	Out.
E.L.	-0.866,	-0.350,	-0.651	Tr.	Out.	Int.
Int.	0.993,	0.879,	0.150	Event	Event	Zero
Tr.	-0.033,	0.839,	0.705	Exp.	Exp.	Event
Exp.	0.223,	0.159,	0.880	M.E.	Ind.	Prb.
M.E.	0.404,	-0.399,	-1.105	Prb.	Tr.	Tr.
Out.	-0.381,	-0.291,	-0.224	Int.	Int.	Exp.

Multidimensional Scaling Solution for Paragraph
Construction Data Eighth Grade Control Subjects

	Final Construction 2 Dimensions	Stress = .060	Sorted Configurations Dimension 1	2
Prb.	-0.709,	-0.965	M.E.	Prb.
Ind.	-0.967,	0.066	Ind.	Zero
Event	-0.098,	0.542	Prb.	Exp.
Zero	0.935,	-0.728	Event	Out.
E.L.	0.176,	-0.186	Tr.	E.L.
Int.	1.447,	1.061	Exp.	Ind.
Tr.	0.058,	0.545	E.L.	M.E.
Exp.	0.060,	-0.423	Out.	Event
M.E.	-1.349,	0.296	Zero	Tr.
Out.	0.448,	-0.209	Int.	Int.

Multidimensional Scaling Solution for Paragraph
Construction Data High School Experimental Subjects

	Final Configuration 3 Dimensions Stress = .036	Sorted Configurations Dimension		
		1	2	3
Prb.	-0.528, -0.454, -0.040	Zero	E.L.	Int.
Ind.	-0.135, 1.149, 0.182	Tr.	Int.	Zero
Event	0.097, -0.486, 0.532	Prb.	Event	Exp.
Zero	-0.565, -0.071, -0.781	Exp.	Prb.	Prb.
E.L.	-0.103, -1.300, 0.204	Ind.	Zero	Out.
Int.	1.101, -0.498, -0.793	E.L.	M.E.	Ind.
Tr.	-0.543, 1.023, 0.345	Out.	Out.	E.L.
Exp.	-0.170, 0.410, -0.278	Event	Exp.	Tr.
M.E.	0.808, -0.021, 0.631	M.E.	Tr.	Event
Out.	0.038, 0.245, -0.003	Int.	Ind.	M.E.

Multidimensional Scaling Solution for Paragraph Construction Data High School Control Subjects

	Final Configuration 4 Dimensions Stress = .075*				Sorted Configurations Dimension			
					1	2	3	4
Prb.	0.117,	-0.443,	-0.276,	-0.835	Event	E.L.	E.L.	Prb.
Ind.	-0.535,	0.912,	-0.402,	-0.659	Ind.	M.E.	Int.	Ind.
Event	-0.638,	-0.590,	-0.033,	-0.215	Zero	Event	Ind.	Exp.
Zero	-0.364,	0.041,	0.744,	-0.151	E.L.	Prb.	Prb.	Event
E.L.	-0.175,	-0.692,	-0.681,	0.388	Tr.	Int.	Tr.	Zero
Int.	0.694,	0.017,	-0.476,	0.230	Prb.	Zero	Event	Out.
Tr.	-0.140,	0.689,	-0.268,	0.437	Out.	Exp.	Exp.	Int.
Exp.	0.483,	0.336,	0.311,	-0.336	M.E.	Out.	M.E.	E.L.
M.E.	0.425,	-0.632,	0.438,	1.122	Exp.	Tr.	Out.	Tr.
Out.	0.133,	0.362,	0.643,	0.019	Int.	Ind.	Zero	M.E.

*Unable to obtain a stress ≤ .060 in 1 to 5 dimensional spaces

Appendix C

Median and Mean Relatedness Coefficient Matrices

The upper triangular portion of each matrix reported in this section is the upper triangular portion of a median RC matrix. The lower triangular portion of each matrix corresponds to the lower triangular portion of a mean RC matrix. (Note that every median/mean RC matrix is symmetric.) Diagonal elements of each median/mean RC matrix are 1.0, but are deleted in this section for clarity of presentation.

Abbreviations used in this Appendix

P. = Probability
I. = Independent
Ev. = Event
Z. = Zero
E.L. = Equally Likely
In. = Intersection
Tr. = Trial
Exp. = Experiment
M.E. = Mutually Exclusive
Out. = Outcome

Sixth Grade Experimental Subjects Pretest

	P.	I.	Ev.	Z.	E.L.	In.	Tr.	Exp.	M.E.	Out.
P.		0	0	0	0	0	0	0	0	0
I.	048		0	0	0	0	0	0	0	0
Ev.	054	063		0	0	0	0	0	0	0
Z.	060	036	044		0	0	0	0	0	0
E.L.	064	034	033	032		0	0	0	0	0
In.	033	045	039	032	024		0	0	0	0
Tr.	058	062	083	053	026	043		0	0	0
Exp.	046	049	074	051	027	038	118		0	0
M.E.	028	045	037	027	024	029	042	033		0
Out.	031	033	035	036	021	035	031	031	033	

Sixth Grade Experimental Subjects Posttest

	P.	I.	Ev.	Z.	E.L.	In.	Tr.	Exp.	M.E.	Out.
P.		0	058	0	0	0	0	0	0	056
I.	122		0	0	0	0	0	0	0	0
Ev.	148	064		0	0	0	0	0	0	154
Z.	078	077	095		0	0	0	0	0	0
E.L.	132	050	092	054		0	0	0	0	0
In.	084	084	066	070	068		0	0	0	0
Tr.	144	078	203	057	059	060		154	0	0
Exp.	114	055	247	069	067	044	291		0	0
M.E.	079	070	075	076	087	077	061	064		0
Out.	169	070	236	094	066	063	141	178	068	

Sixth Grade Experimental Subjects Retention

	P.	I.	Ev.	Z.	E.L.	In.	Tr.	Exp.	M.E.	Out.
P.		0	167	074	058	0	148	222	0	309
I.	197		0	0	0	0	0	0	0	0
Ev.	208	092		0	0	0	241	148	0	185
Z.	130	087	085		0	0	0	0	0	0
E.L.	206	136	156	077		0	0	0	0	0
In.	084	069	099	103	066		0	0	0	0
Tr.	212	130	253	098	143	100		407	0	123
Exp.	216	109	271	081	111	090	362		0	167
M.E.	154	155	100	093	104	086	094	102		0
Out.	289	114	245	101	122	093	200	219	140	

Sixth Grade Control Subjects Pretest

	P.	I.	Ev.	Z.	E.L.	In.	Tr.	Exp.	M.E.	Out.
P.		0	0	0	0	0	0	0	0	0
I.	027		0	0	0	0	0	0	0	0
Ev.	0	002		0	0	0	0	0	0	0
Z.	038	020	010		0	0	0	0	0	0
E.L.	032	015	007	046		0	0	0	0	0
In.	023	022	003	017	010		0	0	0	0
Tr.	008	007	003	012	009	0		0	0	0
Exp.	010	005	031	017	014	002	050		0	0
M.E.	023	030	010	022	008	026	004	017		0
Out.	021	025	013	029	042	004	010	026	021	

Sixth Grade Control Subjects Posttest

	P.	I.	Ev.	Z.	E.L.	In.	Tr.	Exp.	M.E.	Out.
P.		0	0	0	0	0	0	0	0	0
I.	036		0	0	0	0	0	0	0	0
Ev.	005	016		0	0	0	0	0	0	0
Z.	043	017	018		0	0	0	0	0	0
E.L.	027	036	010	030		0	0	0	0	0
In.	038	038	011	026	036		0	0	0	0
Tr.	042	049	020	017	028	070		0	0	0
Exp.	021	042	041	016	031	037	033		0	0
M.E.	054	040	012	026	055	056	049	064		0
Out.	035	018	041	050	020	022	013	041	026	

Sixth Grade Control Ss Retention

	P.	I.	Ev.	Z.	E.L.	In.	Tr.	Exp.	M.E.	Out.
P.		0	0	0	0	0	0	0	0	0
I.	052		0	0	0	0	0	0	0	0
Ev.	039	033		0	0	0	0	0	0	0
Z.	028	024	042		0	0	0	0	0	0
E.L.	005	0	023	031		0	0	0	0	0
In.	020	028	026	028	020		0	0	0	0
Tr.	036	049	035	026	011	068		0	0	0
Exp.	044	047	099	030	020	053	095		0	0
M.E.	040	035	035	028	045	062	051	077		0
Out.	043	035	103	036	023	024	040	082	037	

Eighth Grade Experimental Subjects Pretest

	P.	I.	Ev.	Z.	E.L.	In.	Tr.	Exp.	M.E.	Out.
P.		0	0	0	0	0	0	0	0	0
I.	013		0	0	0	0	0	0	0	0
Ev.	003	014		0	0	0	0	0	0	0
Z.	014	020	015		0	0	0	0	0	0
E.L.	046	016	006	022		0	0	0	0	0
In.	005	007	0	014	008		0	0	0	0
Tr.	005	016	040	002	004	0		0	0	0
Exp.	008	016	043	019	019	0	182		0	0
M.E.	005	012	008	006	053	005	009	010		0
Out.	030	006	014	042	017	006	007	014	002	

Eighth Grade Experimental Subjects Posttest

	P.	I.	Ev.	Z.	E.L.	In.	Tr.	Exp.	M.E.	Out.
P.		136	414	059	179	008	205	231	097	210
I.	184		055	0	005	0	037	032	183	0
Ev.	352	156		0	034	0	414	478	058	261
Z.	111	086	092		0	0	0	0	0	0
E.L.	218	103	141	067		0	0	073	120	0
In.	125	091	086	077	067		0	0	0	0
Tr.	248	111	410	071	138	080		666	066	258
Exp.	273	110	434	099	124	080	606		073	255
M.E.	149	203	172	088	148	131	130	137		028
Out.	249	095	326	124	113	073	276	293	133	

Eighth Grade Experimental Subjects Retention

	P.	I.	Ev.	Z.	E.L.	In.	Tr.	Exp.	M.E.	Out.
P.		168	313	090	239	122	350	319	130	283
I.	206		130	013	064	024	094	132	094	095
Ev.	323	170		062	107	0	463	514	120	520
Z.	160	099	159		039	0	070	076	018	095
E.L.	254	170	176	132		0	056	191	250	114
In.	165	121	125	075	129		036	020	023	0
Tr.	354	162	439	143	145	145		712	081	457
Exp.	337	178	487	167	194	145	623		138	508
M.E.	206	188	204	124	251	139	180	203		075
Out.	314	160	468	192	191	113	408	433	218	

Eighth Grade Control Subjects Pretest

	P.	I.	Ev.	Z.	E.L.	In.	Tr.	Exp.	M.E.	Out.
P.		0	0	0	0	0	0	0	0	0
I.	019		0	0	0	0	0	0	0	0
Ev.	030	016		0	0	0	0	0	0	0
Z.	017	022	017		0	0	0	0	0	0
E.L.	026	023	025	030		0	0	0	0	0
In.	040	011	034	029	018		0	0	0	0
Tr.	033	021	028	009	017	018		070	0	0
Exp.	043	021	035	012	029	019	200		0	0
M.E.	023	026	030	032	012	019	014	023		0
Out.	028	014	032	042	027	024	012	025	030	

Eighth Grade Control Subjects Posttest

	P.	I.	Ev.	Z.	E.L.	In.	Tr.	Exp.	M.E.	Out.
P.		0	0	0	0	0	0	0	0	0
I.	079		0	0	0	0	0	0	0	0
Ev.	098	064		0	0	0	0	051	0	0
Z.	065	076	057		0	0	0	0	0	0
E.L.	095	059	070	070		0	0	0	0	0
In.	086	052	060	054	044		0	0	0	0
Tr.	104	061	141	063	063	071		384	0	0
Exp.	143	077	166	064	060	073	347		0	0
M.E.	105	101	081	066	085	044	070	069		0
Out.	127	069	141	065	072	062	073	104	093	

Eighth Grade Control Subjects Retention

	P.	I.	Ev.	Z.	E.L.	In.	Tr.	Exp.	M.E.	Out.
P.		0	0	0	0	0	0	0	0	0
I.	118		0	0	0	0	0	0	0	0
Ev.	163	105		0	0	0	014	167	0	008
Z.	080	117	068		0	0	0	0	0	0
E.L.	136	083	079	087		0	0	0	0	0
In.	089	066	080	075	062		0	0	0	0
Tr.	138	096	208	069	088	075		317	0	0
Exp.	133	085	216	052	070	053	345		0	0
M.E.	093	124	084	090	116	067	106	063		0
Out.	124	102	171	075	083	069	164	161	092	

High School Experimental Subjects Pretest

	P.	I.	Ev.	Z.	E.L.	In.	Tr.	Exp.	M.E.	Out.
P.		0	0	0	0	0	0	0	0	0
I.	004		0	0	0	0	0	0	0	0
Ev.	0	0		0	0	0	0	0	0	0
Z.	005	007	015		0	0	0	0	0	0
E.L.	144	030	016	019		0	0	0	0	0
In.	009	041	0	014	036		0	0	0	0
Tr.	028	005	040	014	025	0		231	0	0
Exp.	020	010	022	0	011	037	230		0	0
M.E.	0	042	005	0	030	051	004	031		0
Out.	017	005	052	015	005	019	042	041	024	

High School Experimental Subjects Posttest

	P.	I.	Ev.	Z.	E.L.	In.	Tr.	Exp.	M.E.	Out.
P.		059	230	093	252	0	138	183	0	254
I.	085		0	0	0	0	0	0	0	0
Ev.	224	089		0	0	0	278	338	0	278
Z.	122	041	080		058	0	0	0	0	0
E.L.	253	075	098	080		0	0	0	0	0
In.	100	056	042	031	057		0	0	0	0
Tr.	160	052	318	072	061	051		680	0	178
Exp.	254	078	371	052	075	049	589		0	267
M.E.	082	073	034	069	133	041	090	063		0
Out.	236	087	344	101	094	048	228	229	080	

High School Experimental Subjects Retention

	P.	I.	Ev.	Z.	E.L.	In.	Tr.	Exp.	M.E.	Out.
P.		0	166	043	250	0	300	244	0	316
I.	092		0	0	099	0	0	0	0	0
Ev.	292	093		0	133	0	616	593	0	644
Z.	111	026	073		0	0	0	0	0	010
E.L.	191	111	196	036		0	0	0	111	095
In.	048	073	040	051	069		0	0	0	0
Tr.	269	117	532	075	193	045		704	0	504
Exp.	233	116	558	076	172	049	711		0	567
M.E.	097	061	141	060	150	048	119	080		0
Out.	361	140	514	137	227	068	439	506	113	

High School Control Subjects Pretest

	P.	I.	Ev.	Z.	E.L.	In.	Tr.	Exp.	M.E.	Out.
P.		0	0	0	0	0	0	0	0	0
I.	0		0	0	0	0	0	0	0	0
Ev.	0	0		0	0	0	0	0	0	0
Z.	0	021	0		0	0	0	0	0	0
E.L.	041	0	0	0		0	0	0	0	0
In.	016	019	006	010	013		0	0	0	0
Tr.	050	0	0	003	0	009		0	0	0
Exp.	034	0	011	0	006	012	116		0	0
M.E.	0	086	0	0	011	0	0	0		0
Out.	023	0	090	0	031	020	0	040	0	

High School Control Subjects Posttest

	P.	I.	Ev.	Z.	E.L.	In.	Tr.	Exp.	M.E.	Out.
P.		0	0	0	0	0	0	0	0	0
I.	060		0	0	0	0	0	0	0	0
Ev.	102	041		0	0	0	0	0	0	0
Z.	021	020	020		0	0	0	0	0	0
E.L.	119	020	026	010		0	0	0	022	0
In.	0	017	0	028	006		0	0	0	0
Tr.	154	042	098	031	043	0		483	0	0
Exp.	117	048	106	020	046	0	411		0	043
M.E.	025	084	025	033	073	022	035	027		0
Out.	120	037	132	045	060	0	104	111	035	

High School Control Subjects Retention

	P.	I.	Ev.	Z.	E.L.	In.	Tr.	Exp.	M.E.	Out.
P.		0	191	0	0	0	191	191	0	143
I.	029		0	0	0	0	0	0	043	0
Ev.	245	007		0	0	0	311	476	0	283
Z.	046	023	022		014	028	0	012	039	0
E.L.	067	023	079	039		022	0	005	168	0
In.	031	038	006	067	107		0	0	037	0
Tr.	313	0	318	017	048	0		548	0	122
Exp.	263	0	395	035	041	045	487		0	179
M.E.	086	160	090	038	175	096	044	059		0
Out.	254	011	258	006	043	0	240	265	020	